27/03/2019

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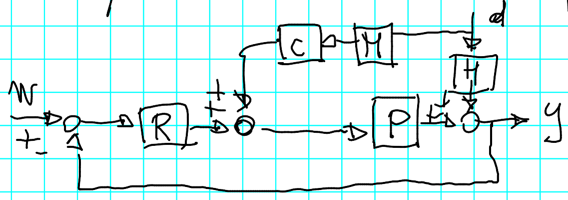
Feasibility (disturbance compensation)



$$\frac{y}{D} = 0 \Rightarrow \quad H + M G P = 0$$

$$G_D = -\frac{H}{MP}$$

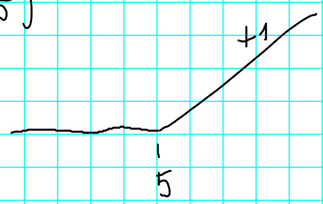
Example

$$P = H = \frac{1}{1 + 10s}$$

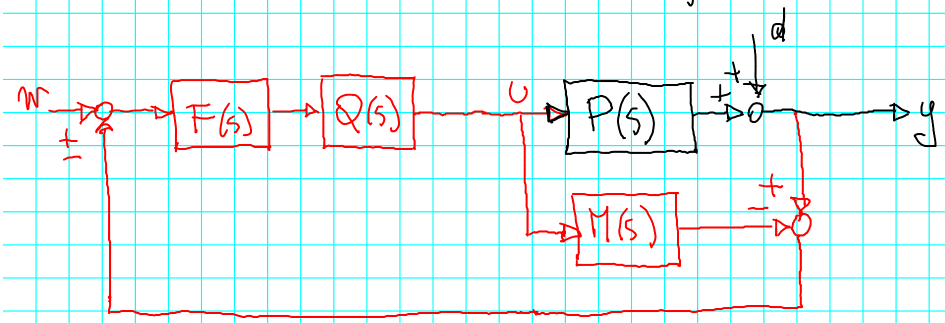
$$H = \frac{1}{1 + 0.2s}$$

$$C_{1D} = -\left(1 + \frac{s}{5}\right)$$

$|C_{1D}|$ 0dB

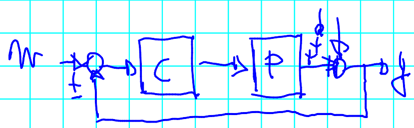


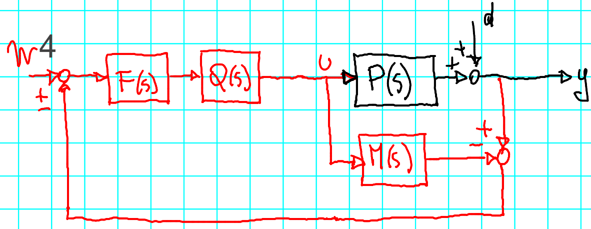
Internal Model Control (IMC) scheme (~ 1974)



$$\Updownarrow$$

$$C = \frac{QF}{1 - QFM}$$





Suppose Nominal Conditions: $\begin{cases} M = P \\ d = 0 \end{cases}$ $\Rightarrow$ there is no loop

$$\Rightarrow \frac{Y}{W} = FQP = FQM$$

Suppose also that you can set $Q = P^{-1} \Rightarrow \dfrac{Y}{W} = F$

5 1) No model errors, no disturbances ⇒ no (need for) feedback

2) $P(s)$ asymp. stable & minimum-phase
⇒ can choose $w \to y$ dynamics arbitrarily provided rel. degree is preserved

Why we talk about IMC

- easy tool to tune PI / PID controllers
- easy to create adaptive controllers (give me a newly identified M ⇒ new C)
- easy to assess tolerance to model errors (~robustness)

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$$P(s) = \mu \frac{e^{-sD}}{1+sT} \qquad T > 0, \; D \geqslant 0 \quad (\mu > 0)$$

$$Q(s) = \frac{1+sT}{\mu}$$

$$F(s) = \frac{1}{1+s\lambda}$$

λ desired closed-loop time constant

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$$C = \frac{QF}{1 - QFM} = \frac{\frac{1+sT}{\mu}\frac{1}{1+s\lambda}}{1 - \frac{1+sT}{\cancel{\mu}}\frac{1}{1+s\lambda}\cancel{\mu}\frac{e^{-sD}}{\cancel{1+sT}}} = \frac{1}{\mu}\frac{1+sT}{1+s\lambda - e^{-sD}}$$

- $e^{-sD} \longrightarrow (1,0)$ Padé $\Rightarrow 1 - sD$

$$C = \frac{1}{\mu}\frac{1+sT}{\cancel{1}+s\lambda \cancel{-1}+sD} = \frac{1}{\mu}\frac{1+sT}{s(\lambda+D)}$$

PI

$$\boxed{T_i = T}$$

$$\frac{K}{T_i} = \frac{1}{\mu(\lambda+D)} \Rightarrow \boxed{K = \frac{T}{\mu(\lambda+D)}}$$

- $e^{-sD} \rightarrow (1,1)$ Padé $\rightarrow \dfrac{1 - sD/2}{1 + sD/2}$

$$C = \frac{1}{\mu} \frac{1 + sT}{1 + s\lambda - \frac{1 - sD/2}{1 + sD/2}} = \frac{1}{\mu} \frac{(1+sT)(1+sD/2)}{(1+s\lambda)(1+sD/2) - 1 + sD/2}$$

$$= \frac{1}{\mu} \frac{(1+sT)(1+sD/2)}{s\left(s\frac{\lambda D}{2} + \lambda + D\right)}$$

Real PID

$\Downarrow K, T_i, T_d, N$

(2) implicit

③ Loop made of C & P (P_n as. stable)

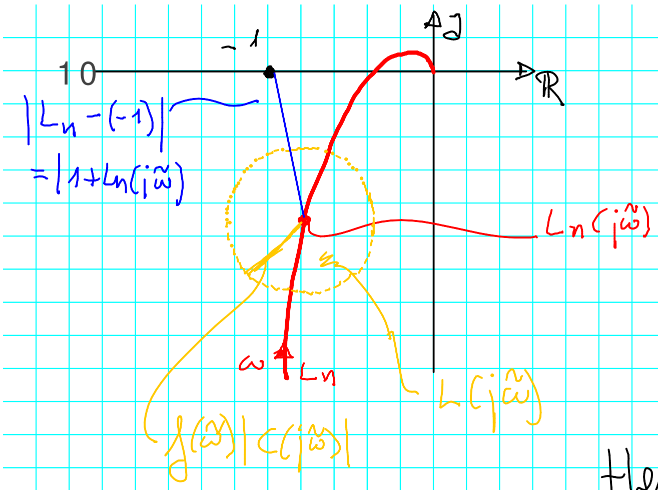
Let $P = P_n + \Delta P$ (ΔP additive model error)

ΔP is unknown BUT we know that

$|\Delta P(j\omega)| < f(\omega) \quad \forall \omega$ (know f)

(technical) P_n & $P_n + \Delta P$ have both no RHP poles

$\Rightarrow$ if the loop with C & P_n is stable, then so is the one with P provided that ΔP does not change the # of turns of the loop freq response around point -1



$$L_n = C P_n$$

$$L = C(P_n+\Delta P)$$

$$= L_n + C\Delta P$$

$$|L-L_n| = |C\Delta P| < |C| f$$

IF $\forall\ \tilde\omega$ we have

$$|C(j\tilde\omega)|\, f(\omega) < |1+L_n(j\tilde\omega)|$$

then ΔP cannot destroy the loop asymp. stability

Hence a sufficient robustness condition is

$$|1+L_n(j\omega)| > |C(j\omega)|\, f(\omega) \qquad \forall \omega$$

which means

$$f(\omega) < \underbrace{\left|\frac{1+L_n(j\omega)}{C(j\omega)}\right|}_{\text{KNOWN from nominal conditions}}$$