05/06/2019

$\boxed{E2}$ 22/06/2018, E2

Body with heat capacity $C = 10\ kJ/°C$

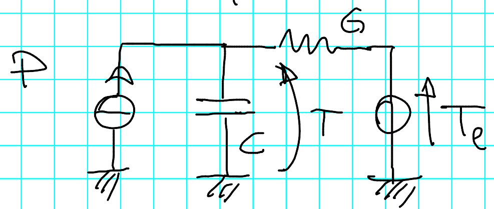
Two daisy-chained heaters, both with max power $P_{max} = 1\ kW$ and 1st order dynamics with time constant $\tau = 5s$. Heat dispersed through a conductance $G = 200\ W/°C$ to an extend fixed temperature T_e.

1) Electric equivalent (consider a single heater of power P)

2) PI scheme representing the daisy chain, tuned for a closed-loop dominant TC of 200 s maximum

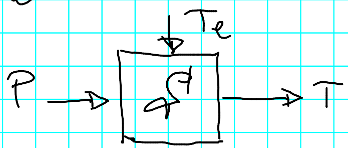
3) obtained φ_m

2) Electric equivalent



P: total heating power

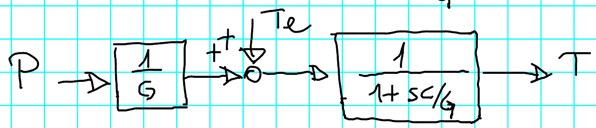
3) Scheme



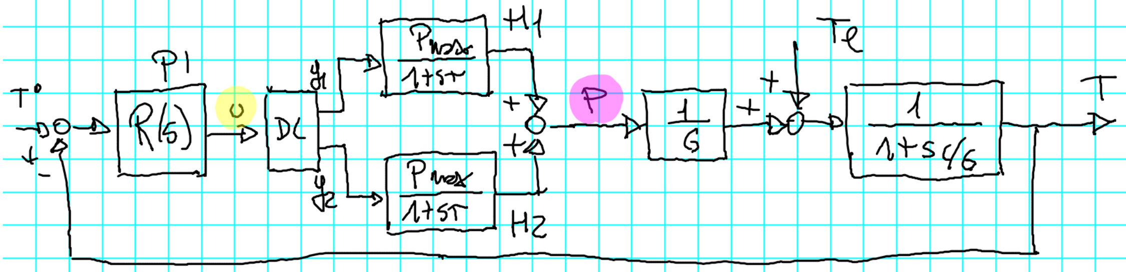
$$C\dot{T} = P - G(T - T_e)$$

$$(sC + G)T = P + GT_e$$

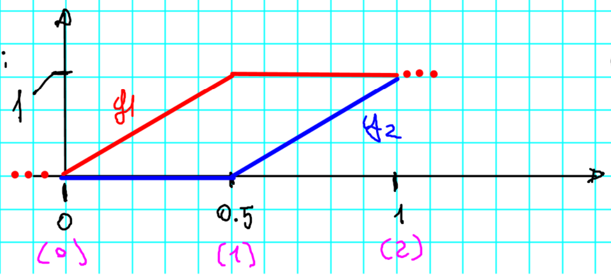
$$T = \frac{1}{sC+G} \cdots = \frac{1/G}{1 + sC/G}\,(P - GT_e)$$



Complete diagram



DC :



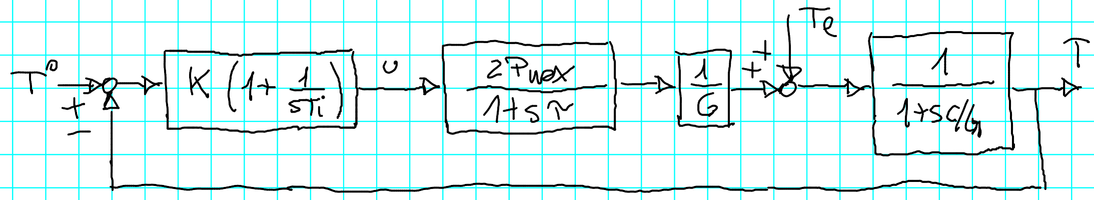
With 0 – 1 , statically

u ∈ [0,1] ⇒ P [0, 2Pmax]

With 0 – 2 statically

u ∈ [0, 2] ⇒ P [0, 2Pmax]

3) Equivalent scheme for tuning



$P_{max} = 1000\,W$

$\tau = 5\,s$ ⇒ 1 decade faster than dominant time constant and also well below the required closed-loop time constant (200s)

$\frac{C}{G} = \frac{10^4}{2\cdot 10^2} = 50\,s$

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- I want $\frac{T}{T^o} \sim \frac{\omega_c}{s}$

But $L(s) = R(s) \dfrac{2P_{max}/G}{(1+s\tau)(1+sC/G)}$

hence with a PI I have to accept a 2nd pole in L (plus that in s=0)

I am cancelling the slower one $(-G/C)$ and leave the other one, so I am prescribing

$$L = \frac{\omega_c}{s(1+s\tau)}$$

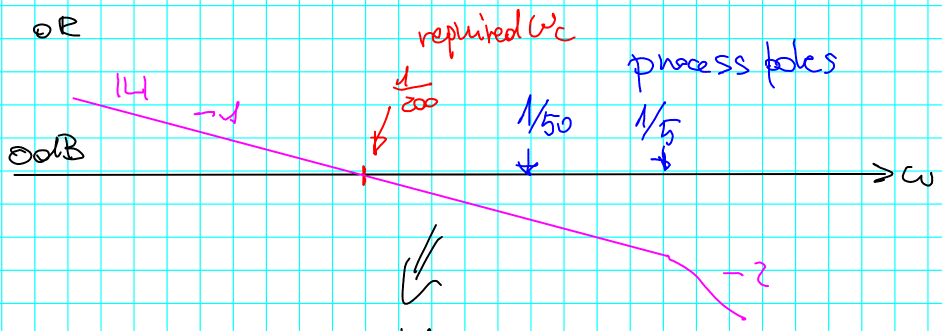
7 Hence $T_i = C/G$ and

$$L(s) = K \frac{\cancel{1+s\,C/G}}{s\,C/G} \frac{2P_{max}}{1+sT} \frac{1/G}{\cancel{1+s\,C/G}}$$

$$= \frac{2P_{max}K/\cancel{G}}{s\frac{C}{\cancel{G}}(1+sT)} = \frac{\boxed{2P_{max}K/C}}{s(1+sT)} \leftarrow \omega_c$$

I need $\frac{2\cdot 1000\,K}{10000} = \frac{1}{200} \Rightarrow K = 0{,}025$

$T_i = 50$



⇓

read L

⇓

compute R

⇓

φ_m with phase rules

3) Phase margin

$$L(s) = \frac{1}{s\,200\,(1+5s)}$$

$\omega_c = \frac{1}{200}$ $\qquad \varphi_m = 90° - \operatorname{arctg}°\left(\frac{5}{200}\right) \ldots$

$\square$

Addendum

H1 & H2 have efficiencies of 0.8 &
0,75 respectively
1 kWh of energy costs 0.3 €
Draw the cost rate curve as a function
of $T - T_e$ at steady state :

1 $1\,kWh = 1000\,W \cdot 3600\,s = 3.6 \cdot 10^6\,J$

$1\,W \rightarrow 1\,\frac{J}{s}$ will cost $\frac{0{,}3\,€/kWh}{3.6 \cdot 10^6\,J/kWh} \leftarrow \frac{€}{J}$

$\rightarrow \frac{0.3}{3.6 \cdot 10^6}\,\frac{€}{s}$

OR $1\,kW \cdot 1\,h \Rightarrow 0{,}3\,€$

$1\,kW \Rightarrow 0.3\,\frac{€}{h}$

Hence a power of 1 kW corresponds to 0.3 €/h

Statically

$$P = G\overbrace{(T - T_e)}^{\Delta T} \qquad P = G\,\Delta T$$

↑ power released to the body

$$P_{\text{taken by heaters}} = \begin{cases} P/0.8 & 0 \le P \le 1000 \\ \dfrac{1000}{0.8} + \dfrac{P-1000}{0.75} & 1000 < P \le 2000 \end{cases}$$

$$\text{Cost rate} = \begin{cases} \dfrac{G\Delta T}{0.8} \cdot 0.3 \ \dfrac{€}{h} & 0 \le G\Delta T < 1 \\ \left(\dfrac{1}{0.8} + \dfrac{G\Delta T - 1}{0.75}\right) \cdot 0.3 \ \dfrac{€}{h} & 1 \le G\Delta T \le 2 \end{cases}$$

P in kW

kW/°C