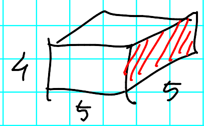
29/05/2019

E1

Body of $5\,m \cdot 5\,m \cdot 4\,m$ extension



density $1.1\ kg/m^3$

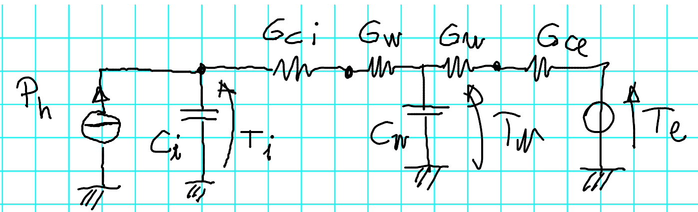
spec. heat $1020\ J/kg°C$

dispersing heat by convection ($\gamma = 8\ W/m^2 °C$) to a wall of surface $5 \cdot 4\ m^2$, thickness 40 cm, density $3000\ kg/m^3$, $c = 800\ J/kg°C$, thermal conductivity $\lambda = 0.8$ (SI units) and then by convection ($\gamma = 4$) to ext. fixed T

Body connected to heater of max power 2 kW

1) Draw an electric equivalent for the physical system
2) Estimate power needed to keep a 20°C difference of internal T wrt ext T
3) Set up a control scheme for the internal (body) temperature acting on the heater; tune this control for a settling time of 10 min
4) Discuss the heater sizing and provide some estimate of how fast it has to respond to its command
5) Discuss the same sizing as for the energy cost of a T set point variation

1)



C_i, C_w	Heat capacities of internal (body) & wall
G_{ci}, G_{ce}	convective conductances (internal & external)
G_w	conductance of 1/2 wall
P_h	heater power
T_i, T_e, T_w	internal, external & wall temperatures

4 $C_i = 5 \cdot 5 \cdot 4 \cdot 1{,}1 \cdot 1020 = 112200 \quad J/°C$

$C_w = 5 \cdot 4 \cdot 0{,}4 \cdot 3000 \cdot 800 = 19200000 \quad J/°C$

$G_{ci} = 5 \cdot 4 \cdot 8 = 160 \quad W/°C$

$G_{ce} = 5 \cdot 4 \cdot 4 = 80 \quad W/°C$

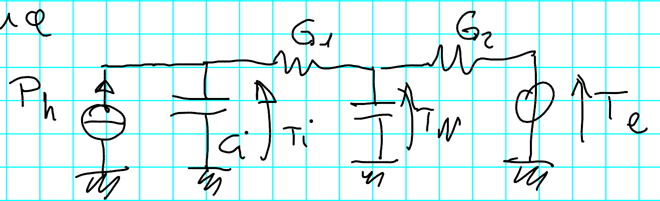
$G_w = 0.8 \frac{5 \cdot 4}{0.2} = 80 \quad W/°C$

Series of G_{ci} & G_w $\Rightarrow G_1 = \frac{1}{1/G_{ci} + 1/G_w} = 53$

$$\frac{1}{G_1} = \frac{1}{G_{ci}} + \frac{1}{G_w}$$

Series of G_{ce} & G_w $\Rightarrow G_2 = \frac{1}{1/G_{ce} + 1/G_w} = 40$

Hence



Dynamic model:

$$C_i \dot{T}_i = P_h - G_1 (T_i - T_w)$$
$$C_w \dot{T}_w = G_1 (T_i - T_w) - G_2 (T_w - T_e)$$

$$\begin{cases} \dot{T}_i = -\dfrac{G_1}{C_i} T_i + \dfrac{G_1}{C_i} T_w + \dfrac{1}{C_i} P_h \\ \dot{T}_w = \dfrac{G_1}{C_w} T_i - \dfrac{G_1 + G_2}{C_w} + \dfrac{G_2}{C_w} T_e \end{cases}$$

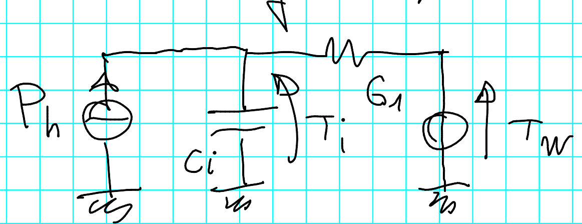
6 $$\begin{bmatrix} \dot{T}_i \\ \dot{T}_w \end{bmatrix} = \begin{bmatrix} \frac{-53}{112200} & \frac{53}{112200} \\ \frac{53}{19200000} & \frac{-93}{19200000} \end{bmatrix} \begin{bmatrix} T_i \\ T_w \end{bmatrix} + \begin{bmatrix} \frac{1}{112200} & 0 \\ 0 & \frac{40}{19200000} \end{bmatrix} \begin{bmatrix} P_h \\ T_e \end{bmatrix}$$

time constants $\sim$ 2100 and 476000 seconds

For controlling internal T $\nearrow$ consider wall T as fixed (in reality it is a VERY slow disturbance when viewed by the internal T loop)

NOTE: cancel too slow a pole
$\Rightarrow$ zero in controller at too slow a frequency
$\Rightarrow$ control kicks
$\Rightarrow$ waste of energy

2) For controlling T_i I consider



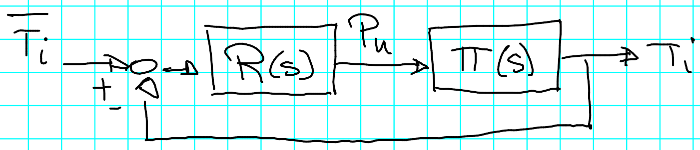
$$C_i \dot{T}_i = P_h - G_1 (T_i - T_W)$$

$$(sC_i + G_1) T_i = P_h + (G_1 T_W)$$

I want to use this for control

$$\frac{T_i(s)}{P_u(s)} = \frac{1}{sC_i + G_1} = \frac{1/G_1}{1 + s\frac{C_i}{G_1}} = \Pi(s)$$

Scheme:

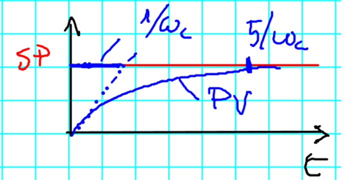


Choice of a structure for R(s) & tuning

$$\left.\begin{array}{l}\text{1st order } \Pi \\ \text{speed spec only} \\ \text{no tight control} \\ \text{(shape of transients specified)}\end{array}\right\} \Rightarrow \text{PI}$$

9 Tuning

Settling time $= \dfrac{5}{\text{cutoff frequency}} \Rightarrow \omega_c = \dfrac{5}{t_{set}}$



Desired loop TF: $\dfrac{\omega_c}{s}$

$$R = \frac{\omega_c/s}{P} = \frac{5}{t_{set}\cdot s}\ \frac{1+s\,C_i/G_1}{1/G_1}$$

integral time τ_i

gain / integral time $\sim \dfrac{K}{\tau_i}$

int. time $\tau_i = \dfrac{C_i}{G_1} = 2610$

gain $\quad K = \tau_i \dfrac{5}{t_{set}} G_1 \doteq 935$

10) When I apply a setpoint step, the control signal responds through the control sens. function

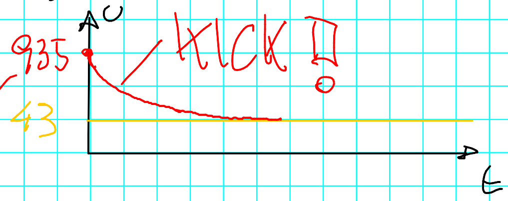
$$\bar{T}_i \longrightarrow \boxed{\frac{R}{1+RP}} \longrightarrow u$$

$$\frac{935\,\frac{1+2610s}{2610s}}{1+935\,\frac{\cancel{1+2610s}}{2610s}\,\frac{1/43}{\cancel{1+2610s}}} =$$

$$\frac{935(1+2610s)}{2610s+\frac{935}{43}} = \frac{43(1+2610s)}{1+120s}$$

$$\frac{U}{T_1} = 43 \frac{1+2610\,s}{1+120\,s}$$

unit step response:



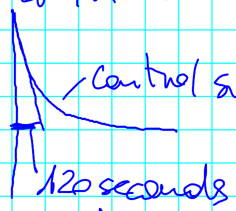
$$43 \, \frac{2610}{120}$$

this kick is inevitable for getting the required t_{set}

However need 935 W of heater to not saturate for a 1°C set point change

IF I WANT that settling time I have to oversize the heater significantly wrt the size needed to sustain the steady state

How fast must the heater be?



its (dominant) TC must be well smaller than 120 s