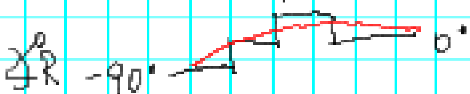
11/06/2014

1

$R(s)$ with pre-specified structure (e.g., PID)

$\Rightarrow$ maximum phase lead ϕ_R

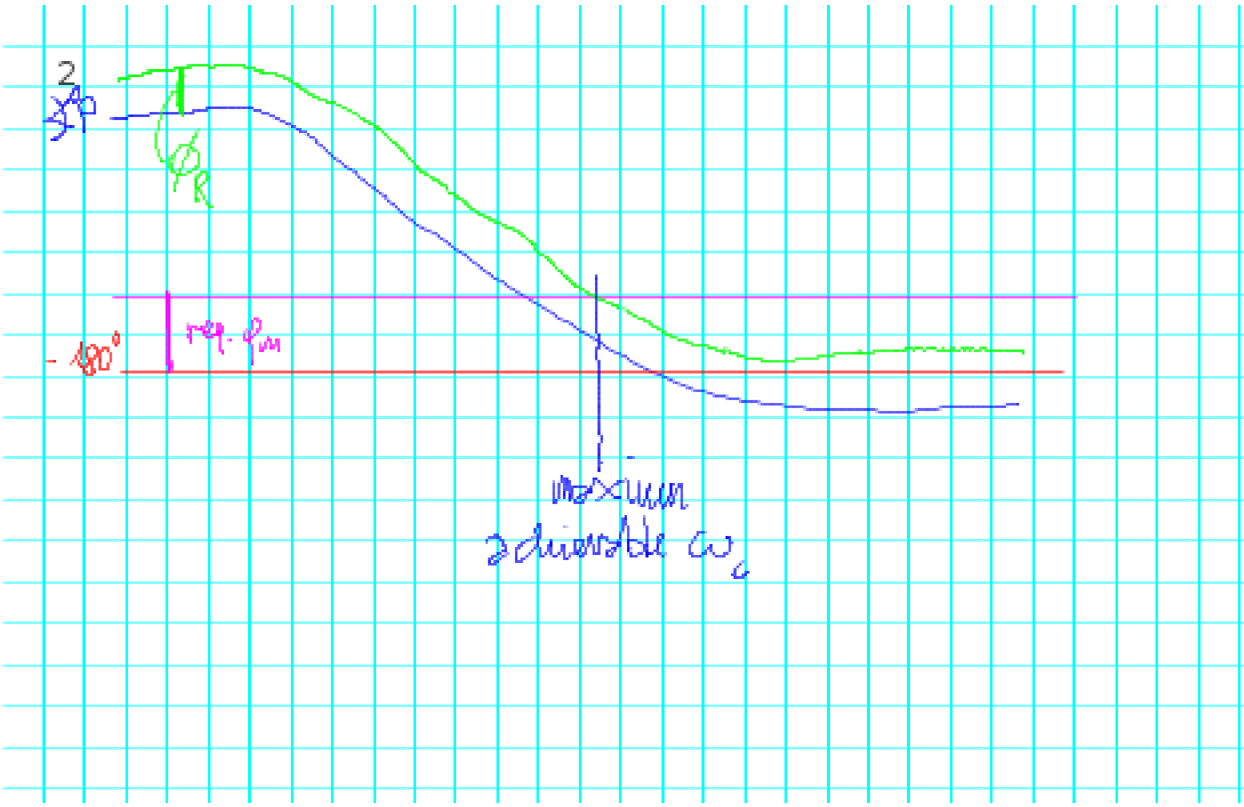
e.g. PID (positive poles)



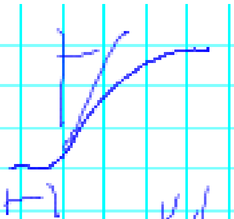
max lead IDEAL $+90°$

REALISTIC $+60°$

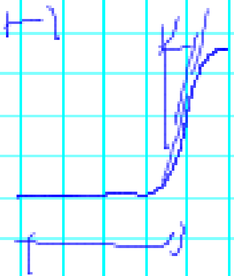
Note: unreviewed material


2
-180°
req. φm
φR
maximum
achievable ωc

$$\frac{K e^{-s\tau}}{1+sT}$$



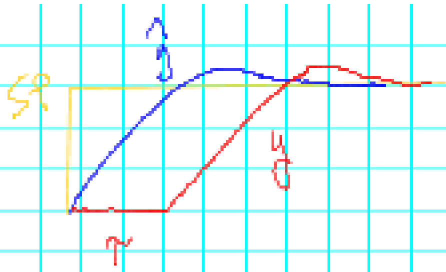
R. Dynamics dominated $\qquad \tau \approx T$



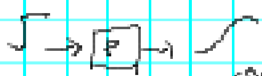
delay dominated $\qquad \tau \gg T$

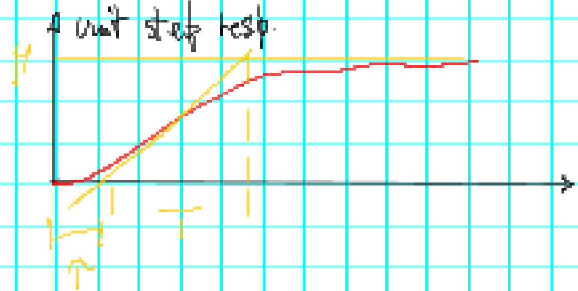
Normalised delay $\theta = \dfrac{\tau}{\tau + T}$

4 S Trobdrr



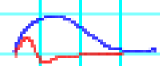
例 C-PID tuning





$$H(s) = \frac{Ke^{-s\gamma}}{1+sT}$$

First
Order
Plus
Dead
Time



IMC :

$$M = \mu \frac{e^{-s\tau}}{1+sT} \qquad Q = \frac{1+sT}{\mu} \qquad \begin{matrix} T>0 \\ \tau>0 \end{matrix}$$

$$F = \frac{1}{1+s\lambda}$$

$$\Rightarrow R = \frac{QF}{1-QFM} = \frac{\frac{1+sT}{\mu}\,\frac{1}{1+s\lambda}}{1-\frac{\cancel{1+sT}}{\cancel{\mu}}\,\frac{1}{1+s\lambda}\,\cancel{\mu}\,\frac{e^{-sT}}{\cancel{1+sT}}} =$$

$$= \frac{1}{\mu}\,\frac{1+sT}{1+s\lambda-e^{-s\tau}}$$

(Dahlin 1968(?))

Padé $(1,0)$ $\Rightarrow$ $e^{-s\tau} \approx 1 - s\tau$

$$R(s) = \frac{1}{\mu}\,\frac{1+s\tau}{\cancel{1}+s\lambda - \cancel{1}+s\tau} = \frac{1}{\mu}\,\frac{1+s\tau}{s(\lambda+\tau)}$$ PI

$$R(s) = K\,\frac{1+sT_i}{sT_i}, \quad T_i = \tau$$

$$\frac{1}{\mu(\lambda+\tau)} = \frac{K}{T_i} \Rightarrow K = \frac{\tau}{\mu(\lambda+\tau)}$$

Tuning rules

Padé (1,1) $\quad e^{-s\tau} = \dfrac{1 - s\tau/2}{1 + s\tau/2}$

$$R(s) = \frac{1}{\mu} \frac{1+sT}{1+s\lambda - \frac{1-s\tau/2}{1+s\tau/2}} = \frac{1}{\mu} \frac{(1+sT)(1+s\tau/2)}{(1+s\lambda)(1+s\tau/2) - 1+s\tau/2} =$$

$$= \frac{1}{\mu} \frac{(1+sT)(1+s\tau/2)}{\cancel{1}+s\lambda+s\tau/2+s^2\lambda\tau/2 \cancel{-1}+s\tau/2} =$$

$$= \frac{1}{\mu} \frac{(1+sT)(1+s\tau/2)}{s\left((\lambda+\tau)+s\lambda\tau/2\right)}$$

Real PID

$$R = K\left(1 + \frac{1}{sT_i} + \frac{sT_d}{1+sT_d/N}\right)$$

Nice fact: N too is computed

IDEAL PID $\quad R_{ID} = K \frac{1 + sT_i + s^2 T_i T_d}{sT_i} = K\left(1 + \frac{1}{sT_i} + sT_d\right)$

REAL PID

$$R_R = K\left(1 + \frac{1}{sT_i} + \frac{sT_d}{1 + sT_d/N}\right) = K \frac{sT_i + s^2 T_i T_d/N + 1 + sT_d/N + s^2 T_i T_d}{sT_i\,(1 + sT_d/N)}$$

$$= K \frac{1 + s\,(T_i + T_d/N) + s^2 T_i T_d\,(1 + 1/N)}{sT_i\,(1 + sT_d/N)}$$

BTW $\lim_{\omega \to \infty} |R(j\omega)| = K(1+N)$