Automation of Energy Systems

Master of Science in Automation Engineering

Alberto Leva

Dipartimento di Elettronica, Informazione e Bioingegneria, Politecnico di Milano
leva@elet.polimi.it

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Lecture 1 (2L)

Course introduction and overview
(general concepts on energy systems and their main functionalities)

Some (bare essential) definitions

A first glance at control problems

Recap, needs and next steps

Course introduction and overview

Foreword and *rationale*

- Systems that produce, distribute and use energy are becoming more and more complex and articulated:
  - different sources (renewable or not);
  - different types of generation (e.g., centralised vs. distributed);
  - complex markets in rapid evolution;
  - ...
- Therefore, energy system experience an increasing need for automation
  - at more and more levels (from the power plant to the town grid, down to the single house);
  - more and more integrated (e.g., to coordinate the generation and use of electricity and heat);
  - and for new demands (comfort, economy, environmental impact,...).

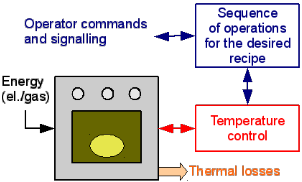
Foreword and *rationale*

- Purpose of the course:
  - address the *scenario* just sketched
    - providing the student with a *system-level* view – typical of the Automation Engineer – on the encountered control problems, the solutions adopted for them to date, and the possible future developments
    - avoiding details on the various types of generators, utilisers and so forth—a matter to which specialised courses are devoted.
- *Caveat emptor*:
  - An exhaustive treatise of the matter is absolutely impossible, even at quite high and abstract a level;
  - thus we shall proceed by introducing general concepts and then going through a few case studies.
  - It will be the duty of *you engineers* to abstract, generalise and transpose the lessons learnt wherever the underlying concepts will need putting at work.

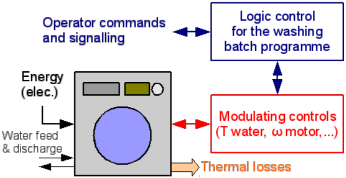
Where is automation in energy systems?

Let us analyse some introductory schemes

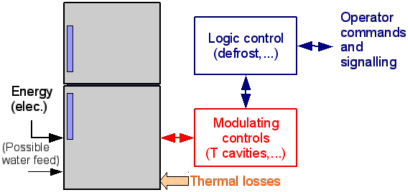
OVEN



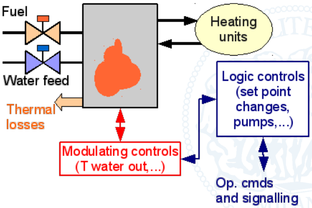
WASHING MACHINE



REFRIGERATOR/FREEZER



WATER BOILER



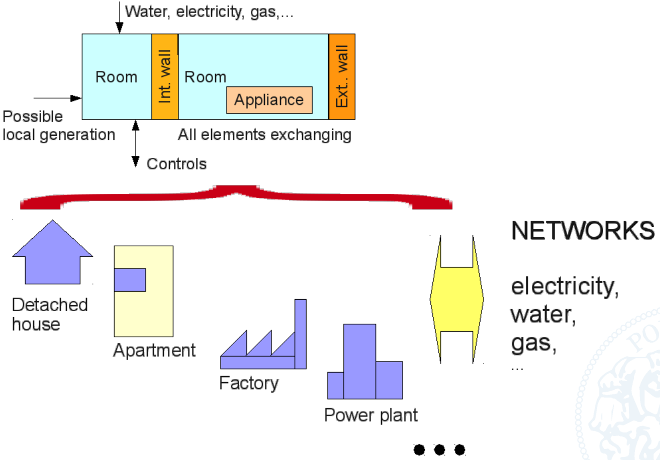
Where is automation in energy systems?

A bit of generalisation

- Carrying on with the list:
  - other generators (solar, wind, geothermal,...);
  - other "utilisers" (heating elements, fan coils,...);
  - maybe a home/building automation system, not installed (only) for energy purpose but surely with a relevant energy effect;
  - ...
- Scaling up:
  - "larger" components (building or compound-level HVAC,...);
  - industrial machines/installations;
  - power plants;
  - ...

Where is automation in energy systems?

Let us now start *aggregating*



NOTE: each part of the system has its own controls and must somehow coordinate with the others.

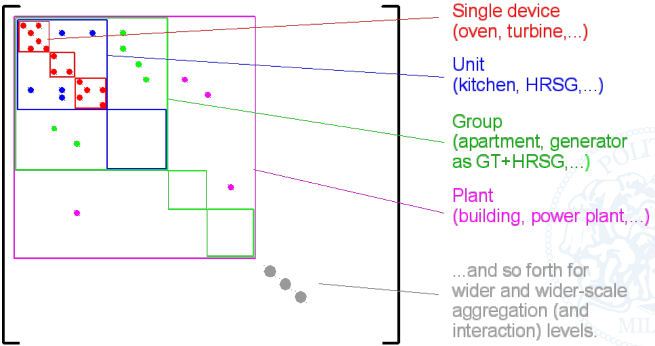
Summing up...

Where is automation in energy systems?
Everywhere:

- everything *transforms* energy
- although – depending on the *main* purpose of each object – the verb "transform" gets specialised for that object as, for example, "generate", "transport", "distribute", "consume" (in the end, such specialisations are little more than conventional)
- and each object (or aggregate of objects) can be controlled in a view to achieving *local* goals (cook some food as quickly as possible; maximise the economic revenue of a household photovoltaic generator based on forecasts of weather, energy use and prices of electricity and gas; generate the total power required at any time by the national network while maintaining each plant as close as possible to its optimal operating point and without overloading the transmission lines;...)
- bearing however in mind that any action on that individual object will (more or less) influence the overall system.

Is there any *hierarchy* in energy ststems?

- Rigorously, no. However such a position, albeit formally correct, is extremely naïve from the engineering point of view
- To understand that, let us see what happens if we attempt to write the whole dynamic model of the "world" energy system (which is *plain crazy*, beware) and observe its *incidence matrix* (a boolean matrix showing which variables appear in which equations):



Why is this naïve?

- Because only a fool could think of a "Giga-controller" doing the entire job:
  - even if one could undertake such a design (just think of how measurements could be collected...)
  - and admitting (we may be crazy, but up to a point) that each component comes with "some control already installed": as a proof of limited craziness, nobody would dismantle the thermostat of each household fridge ;-)
  - the problem would be
    - enormous,
    - of variable structure (components are installed, turned on and off, and some day disposed of),
    - where objectives are decided at various levels, in a time-varying and often conflicting manner (we all cook our dinner in the evening while the gas supplier would like a flat delivery profile),
    - involving physically heterogeneous objects,
    - designed with "standard" specifications and then installed in extremely different conditions (the same air conditioner model can be used in a dining room in town, in a mansard in the mountains, in a tropical beach bar,...),
    - with local controllers (see above) generally designed without having in mind any communication (let alone cooperation) among them,
    - ...

Consequences

- Therefore, some hierarchy - or better, as we shall see, some *structuring* of the problems is in order.
- In other words, we need
  - to understand which are the relevant problems (a task already carried out, although the matter is continuously evolving, think e.g. of environmental issues)
  - and that can reasonably be dominated (which is often not trivial),
  - find for them solution that are sound from an engineering standpoint
  - and figure out how to put said solutions at woke also when the boundary conditions for the problem at hand vary, which can also be due to how the *rest* of the system is affected by the introduced solutions.

Consequences

- What are we meaning for "a problem can be dominated"?
- Essentially that once said problem is stated in system-theoretical terms
  - having extended its size (i.e., the set of described phenomena) enough for the "rest of the world" to be properly represented by boundary conditions and/or disturbances (i.e., exogenous entities)
  - it is possible to find for it a solution of acceptable complexity and implementable with information available in practice.
- An important concept in this regard is that of "dynamic separation", on which we shall return in due time.

Consequences

- To state and address problems this way, we need a "systemic" approach, in which components
  - can be described at different detail levels,
  - but preserving their interfaces with other components of the overall system,
  - and as independently as possible of how they are connected to the rest of the system;
- at the same time, the approach should allow to state problems in such a way to be tractable with well established control methodologies and techniques (although the energy context has been fostering new ones).
- In addition, simulation techniques play a crucial role.

A couple of words on history

- How has automation in energy systems developed?
- Initially (and trivially) to be able to run them: without control systems one can
  - neither operate a power plant
  - nor avoid the electric network collapse
  - nor maintain the required pressures in a gas network extending for tens of thousands of kilometers;
- then (and by the way almost immediately) for efficiency reasons concerning "big" system components (e.g., power plants),
- but always having in mind, more or less explicitly, a fairly well-defined structure of the system (in the electric case, for example, a few large generators, transmission, distribution, and many utilisers of variable size but small if compared to the generators).

A couple of words on history

- Today, however, the *scenario* has changed:
  - there is more and more distributed generation to be integrated with the network,
  - different energy sources, both renewable and not, are being considered,
  - economies of scale are being exploited (think e.g. of district heating),
  - integration is spreading out among controls (e.g., feedforward compensation from heat-releasing appliances to room temperature controllers)...
  - and sometimes among "machines" (e.g., heat recovery, generation surplus management, and so forth).

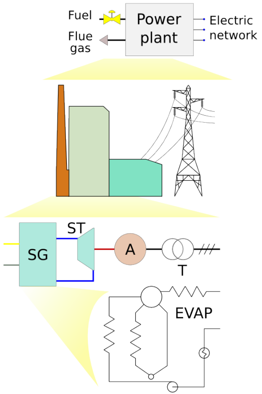
Summing up...

- Energy systems provide a number of control problems to address, and several are "new".
- Many solutions that were acceptable in the past are no longer acceptable today (efficiency demands are becoming more stringent).
- In the course are we thinking to address everything we have mentioned so far?
- Not at all: it would take too long and would not even be a "smart" idea.
- Let us therefore review the initial statement about the purpose of the course, as we can now give it a more precise meaning...

Summing up...

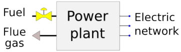
- ...by means of another scheme:

(1) Complex models, typically object-oriented,
for component-level studies



Up to many thousands of equations
and state variables

(2) Simple models, typically object-oriented,
for system-level studies



Up to around ten equations
and state variables

(3) Simple models, typically block-oriented,
for control synthesis

Generally not more than four-five equations
and state variables

- (1) and (2) have the same interfaces;
- (1–3) must be consistent;
- in this course we concentrate on (2–3) and only sketch out relationships with (1).

- The course consists of
  - about 30 hours of lectures
  - and about 20 hours of classroom practice,

  for a total of 5 CFU.
- Lectures are guided with slides,
- while classroom practice involves guidance and individual work.

- Introduction (this lecture);
- review of the mathematical, modelling and control principles that will be used later on;
- the main physical objects (generators, utilisers, distribution networks) involved in energy systems:
  - synthetic description and dynamic behaviour in the context where they operate,
  - simple models, parametrisable with the minimum necessary information;
- the main control problems in energy systems and the strategies to handle them.

Classroom practice sessions, interlaced with lectures, contain examples and case studies concerning small applications, and involve the use of open source simulation and control synthesis tools.

Instructor information and course site

- Alberto Leva
  Dipartimento di Elettronica, Informazione e Bioingegneria
  Politecnico di Milano
  Voice (39) 02 2399 3410
  E-mail alberto.leva@polimi.it

- Office hours:
  - at the DEIB (2nd floor, room 234) on Thursday, 13.30 to 15.30,
  - or by appointment.
- Course web page:
  http://home.deib.polimi.it/leva/CourseAES

Teaching material

- Course slides (*in fieri*);
- some literature and web references, that will be introduced later on.

Software

- **Scilab** (open source, Scilab license):
  analysis, synthesis and simulation of causal dynamic systems (block-oriented approach).

  Info and download: `http://www.scilab.org`
- **OpenModelica** (free software, BSD license):
  creation and simulation of a-causal dynamic systems (object-oriented approach).

  Info and download: `http://www.openmodelica.org`
- **wxMaxima** (free software, GPL license):
  CAS (Computer Algebra System) for symbolic computations.

  Info and download: `http://www.wxmaxima.org`

- **Important note**:
  the goal of this course is not to teach (let alone compare) approaches to modeling and simulation, nor is it to train students for the use of one or another software; the mentioned applications are here *only* "service tools" to put the concepts learnt to work.

Exam

- The students will be required to carry out a project by using the presented methodologies and tools, and to deliver a short written report following a template that will be provided during the course together with the project themes.
- This will contribute about 40% of the total score, the rest coming from a written test (about 1.5 hours) examples of which will be shown later on.
- For apparent reasons, to pass the exam the written test must reach a sufficient score.
- There are regular exam sessions in the periods established by the School.

Some (bare essential) definitions

Energy sources classification

- **Primary – Secondary Energy (PE–SE)**
  - Primary energy is that gathered directly in nature:

    coal, oil, natural gas, biomass, radioactive substances, geothermal energy, wind, sun radiation, gravitational potential energy (e.g. for hydroelectric generation),...
  - Secondary energy is obtained by transforming primary energy in a form easier to use/store/transmit:

    electricity, fossil fuels, hydrogen, steam...
- **Renewable – Non Renewable Energy Sources (RES–NRES)**
  - Renewable energy is obtained from (practically) inexhaustible sources (and without pollutant release):

    sun, wind, tides, geothermal heat,...
  - Non renewable energy implies consuming some "fuel" (and releasing some pollutant):

    fossil fuels,...

$\Longrightarrow$ Extracting, transforming and transmitting energy are industrial processes where automation is required.

Energy intensity, conservation, efficiency

- **Energy intensity**
  Amount of energy per unit of intended (i.e., useful) result.
  - EI of a country: energy consumption per GDP unit;
  - EI of a product: (average) energy consumption per product unit;
  - EI of a service: (average) energy consumption per served request;
  - ...
- **Energy conservation**
  Reducing the (growth of) energy consumption in absolute physical terms.
- **Energy efficiency**
  Reducing the EI of a product/service/whatever while preserving the intended result (e.g., less energy for HVAC with the same comfort).

$\Rightarrow$ Conservation and efficiency are separate but intertwined concepts, and automation (to say nothing of process/control co-design) is useful to pursue them.

A first glance at control problems

Some control problems – fundamental

- **Generator control (PE→SE)**
  - Minimise fuel consumption (NRES) – maximise caption (RES),
  - that is, stay in the vicinity of "optimal" operating points.
- **Utiliser control (SE→final use)**
  - Maintain functional quality (room temperatures, correct appliance operation,...).
- **Transmission control (one type of SE)**
  - Maintain generation–demand balance;
  - Manage storages if applicable;
  - Maintain network operation quality (voltage and frequency, gas pressures,...);
  - Minimise network losses;
  - Avoid network overloading.

Some control problems – advanced

- **Generator energy mix control (NRES)**
  - Mix PEs (e.g., oil and gas)
  - to fulfil SE demands (e.g., electricity and steam)
  - optimising for cost, pollutant emission, or any combination thereof.
- **Utiliser energy mix control**
  - Mix available SEs (e.g., electricity, gas and solar heat stored as hot water)
  - to fulfil final use needs (e.g., electric and thermal loads of a house)
  - optimising for any Key Performance Indicator (KPI) related to the utiliser process.
- **Zone- (or neighbourhood-)level energy mix control**
  - Given a zone with various generators (both RES and NRES) and utilisers,
  - find the optimal management of each generator and utiliser
  - and also of storages where applicable
  - optimising for...? Defining zone-level KPIs is an open problem and often involves conflicts among stakeholders' interests (e.g., we all cook dinner in the evening but gas suppliers like flat demand profiles).

Control problems – main common characteristics

- Set point tracking and/or functional minimisation;
- set point profiles generation;
- scheduling (of generation, utilisation,...);
- disturbance rejection;
- uncertainty (e.g., demand forecast errors, time-varying costs,...);
- hard constraints (e.g., generator operational limits,...);
- soft constraints (e.g., regulations/agreements on acceptable transient overloads, reserve management,...);
- robustness versus plant–model mismatches (e.g., line impedance variations owing to weather conditions,...);
- fault tolerance (e.g., in the case of generator breakdowns).

Control problems – major techniques

- Classical continuous-time control (single- and multi-variable);
- optimal, robust and predictive control (single- and multi-variable);
- control of variable-structure and/or switching systems;
- dynamic optimisation (often for large-scale systems);
- classical discrete-event control;
- supervisory control.

Recap, needs and next steps

Recap

- Automation in energy system means controlling generation, transmission, storage and utilisation...
- ...in a coordinated manner
- and for multiple objectives at different system levels.

- Control problems call for a coordinated use of various techniques
- and solutions need implementing in heterogeneous architectures.

- Process/control co-design is helpful wherever applicable
- but refurbishing of already designed systems is of paramount relevance.

Needs

- A modelling framework;
- quantitative performance indicators (automation-oriented KPIs);
- a problem taxonomy;
- best practices and control design guidelines based on the above,
- and corresponding validation/assessment methodologies;
- awareness of implementation-related facts.

Next steps

- Review modelling principles;
- define a modelling framework suitably managing components' behaviours and interfaces in both the object-oriented (OO) and the block-oriented (BO) context, conceptually relating the two;
- devise the required component models (of course we shall not exhaust this matter) and the major KPIs;
- learn to create system-level compound models tailored to handling problems that involve those KPIs.

Modelling principles
(balance equations and their relevance for the course)

Homework

Modelling principles

Foreword (1/2)

- Control problems are classically divided in "process" and "motion" ones.
- In this course, the first type dominates.
- Furthermore, Partial Differential Equation (PDE) systems are seldom encountered, and never with more than one spatial coordinate. This is the case of distributed-parameter systems (e.g., tubes).
- In such cases, we shall obtain Ordinary Differential Equation (ODE) systems by managing spatial discretisation via the Finite Volumes (FV) approach. This will provide corresponding concentrated-parameter systems.
- We shall encounter physical phenomena of basically three types, namely hydraulic, thermal, and electrical.
- Mechanical phenomena will appear, but sporadically and in one coordinate (when dealing with rotating masses).
- However, most problems are *multi-physics* (i.e., comprise more than one type of phenomena).

Foreword (2/2)

- We therefore need
  - mass, energy and momentum equations for thermo-hydraulic networks (for space reasons we only treat incompressible fluids with a single species);
  - energy equations for solid bodies such as walls (air in buildings is treated the same way via some simplifications that will be introduced in due course together with their validity limits);
  - (semi)empirical equations for heat transfer;
  - equations for electric networks (phasor-based, reduced to a single phase since this simplification does not hinder the explanation of the required concepts);
  - energy equations for rotating masses (e.g., in alternator-based electric generators);
- Of course we deal with *dynamic* balance equations.

- The equation is written with reference to a *control volume*.
- Let M be the (incompressible, single species fluid) mass contained in the volume.
- Let w_i, $i = 1 \ldots n_m$, be the n_m mass flowrates exchanged by that volume with the external environment, considered positive if entering the volume.
- The equation then simply reads

$$\frac{dM(t)}{dt} = \sum_{i=1}^{n_m} w_i(t)$$

where t is obviously the (continuous) time.

- Also this equation is written with reference to a control volume.
- Consider a fluid volume containing a total energy E and where n_m mass flowrates w_i and n_h heat rates Q_j enter (or exit if negative).
- Clearly, the time derivative of E è is the sum of the heat rates (or thermal powers) Q_j, not associated to any mass transfer, and of the energy contribution yielded by the mass flowrates.
- The latter contributions are of two *coexisting* types:
  - (signed) heat transfers inherent to mass transfers, taking the form

$$\text{mass flowrate} \times \text{efluid specific energy} \quad ([kg/s] \times [J/kg] = [J/s] = [W])$$

  - and work exerted by the entering fluid on that contained in the volume or *vice versa*, that in differential and specific form is expressed as

$$d\mathcal{L} = d(pv) = d(p/\rho)$$

    where v is the specific volume and ρ the density. Note that $d\mathcal{L} = pdv + vdp$, where pdv is "compressing work" and vdp "impelling work" (both of course signed).

- Thus, the thermodynamic variable characterising the energy contribution of an entering (or exiting) mass flowrate to a volume, is the fluid specific enthalpy, expressed as
$$h = e + \frac{p}{\rho}$$
where e is the specific internal energy.
- The energy equation then takes the form
$$\frac{dE(t)}{dt} = \sum_{i=1}^{n_m} w_i(t)h_i(t) + \sum_{j=1}^{n_h} Q_j(t).$$
- For incompressible fluids, at pressures and temperatures of interest for this course, the thermal contribution e invariantly dominates the work contribution p/ρ; hence, *in these conditions* one can consider both the specific enthalpy and the specific internal energy to equal cT, where c is the fluid specific heat (assumed here constant) and T its temperature.

- Furthermore here we only deal with single-species fluids, thereby having always to do with a single specific heat c.
- Given all the above, for our purposes the energy equation for fluids is

$$c\frac{dM(t)T(t)}{dt} = c\sum_{i=1}^{n_m} w_i(t)T_i(t) + \sum_{j=1}^{n_h} Q_j(t).$$

where T is the control volume temperature, assumed uniform in accordance with the adopted concentrated-parameter approach, and M the fluid mass.

- Sometimes the mass is constant, like in a tube section always filled with fluid, and thus

$$cM\frac{dT(t)}{dt} = c\sum_{i=1}^{n_m} w_i(t)T_i(t) + \sum_{j=1}^{n_h} Q_j(t).$$

- In other cases, such as tanks, this is not true. It is then convenient to expand the derivative on the left hand side, and subtract the mass equation multiplied by cT. This provides

$$\begin{array}{ccccccc}
cM(t)\frac{dT(t)}{dt} & + & cT(t)\frac{dM(t)}{dt} & = & & c\sum_{i=1}^{n_m} w_i(t)T_i(t) & + & \sum_{j=1}^{n_h} Q_j(t) \\
 & - & cT(t)\frac{dM(t)}{dt} & = & - & cT(t)\sum_{i=1}^{n_m} w_i(t) & & \\
\hline
cM(t)\frac{dT(t)}{dt} & & & = & & c\sum_{i=1}^{n_m} w_i(t)(T_i(t)-T(t)) & + & \sum_{j=1}^{n_h} Q_j(t)
\end{array}$$

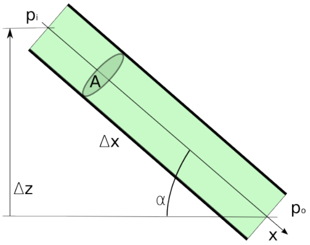
that is sometimes called the "net energy" equation.

- This equation is used primarily for modelling tubes (valves are a somehow analogous case treated later on).
- Consider a tube (element) and write that the time derivative of the fluid momentum is the sum of the forces acting on it, that is,
  - pressure forces at the two ends,
  - gravity force,
  - and friction force on the lateral surface,

  all projected onto the tube *abscissa* x
- In fact, other components do not act on the fluid motion on the *prevailing dimension* x and merely result in constraint reaction forces, not relevant for the energy-related aspects on which we want to focus.

- To keep complexity at a level compatible with the course, consider a tube with uniform section A (and recall that we only deal with incompressible fluids).
- This yields

$$M\frac{du(t)}{dt} = Ap_i(t) - Ap_o(t) + Mg\sin(\alpha) - f_a(t).$$



- Note that $\sin(\alpha) = \Delta z/\Delta x$, Δz being the initial altitude minus the final one
- while the friction force f_a, always contrasting motion, is

$$f_a = K_f A_\ell \rho u|u|$$

where A_ℓ is the lateral surface.

- The quantity K_f is called *friction coefficient*, depends on the fluid/wall contact characteristics, and is tabulated for most cases of interest based on experiments and empirical correlations.
- The equation obviously contains an inertia term, that in our models can however be omitted.
- This is possible because hydraulic phenomena are much faster than thermal ones, which are our main subject.
- In other words, since thermal variables (such as temperatures) propagate at the fluid speed, while hydraulic ones (such as pressures and flowrates) propagate at the speed of sound in the fluid, we can safely assume that for our purposes "hydraulics is always at steady state".
- Thus, we can write the momentum equation as the algebraic one

$$A\left(p_i(t) - p_o(t)\right) + Mg\frac{\Delta z}{\Delta x} - K_f A_\ell \rho u(t)|u(t)| = 0.$$

- Summing up, denoting
  - by A the tube (uniform) section,
  - by L its length,
  - and by ω its internal perimeter,

  simplifying the notation a bit and recalling that $w = \rho A u$, we have

$$A(p_i - p_o) + \rho A L g \frac{\Delta z}{L} - K_f \omega L \rho u|u| = 0$$

$$p_i - p_o = K_f \frac{\omega L}{\rho A^3} w|w| - \rho g \Delta z.$$

- In addition, if the tube is installed in such a way that w has always the same sign, taken positive when going from the higher- to the lower-pressure end, we can write

$$p_i - p_o = K_f \frac{\omega L}{\rho A^3} w^2 - \rho g \Delta z,$$

  that we shall often further summarise as

$$p_i - p_o = \frac{K_T}{\rho} w^2 - \rho g \Delta z.$$

- The momentum equation can also represent the typical valve behaviour.
- This means neglecting more than one phenomenon, that however is of interest only for sizing, or operating conditions not advised for "good" plant management—i.e., not of interest here.
- From our point of view, consider a valve like a variable-section short tube installed so that the flow does not reverse. Thus, take

$$p_i - p_o = \frac{K_T}{\rho} w^2 - \rho g \Delta z,$$

set $\Delta z = 0$ (short component, hardly any gravity effect) and rewrite as

$$w = C_v \Phi(x) \sqrt{\rho(p_i - p_o)}$$

where C_v is the *flow coefficient*, $x \in [0, 1]$ the command, and $\Phi(x)$, $\Phi(0) = 0, \Phi(1) = 1$ the *opening* or *intrinsic characteristic*.

- In this case we naturally neglect any volume effect, and consider the specific heat spatially uniform (non-homogeneous walls will be treated with at least one equation per material layer).
- Since there is no mass transfer, the equation simply reads

$$cM\frac{dT(t)}{dt} = \sum_{j=1}^{n_h} Q_j(t).$$

where symbols have the same meaning as in the fluid case, and M is of course constant.

Heat transfer equations

Foreword

- These are algebraic equations, as they describe no storage.
- We need to model
  - conduction within solids (and sometimes fluids),
  - convections between a solid and a fluid,
  - and radiation.
- In all cases we shall adopt simplified concentrated-parameter descriptions right from the beginning.

- We shall only use a simplified planar descriptions, as more detailed ones would stray from our scope.
- The heat rate from the a to the b side of a solid layer is

$$Q_{ab} = G(T_a(t) - T_b(t))$$

  where T_a and T_b are the side temperatures.
- The thermal conductance G is

$$G = \lambda \frac{A}{s}$$

  where λ is the material's thermal conductivity, A the layer surface, and s its thickness.

- The convective heat rate from a solid wall (subscript w) to a fluid (subscript f) is

$$Q_{wf} = \gamma A(T_w(t) - T_f(t))$$

  where T_w and T_f are the wall and a fluid "bulk" temperature, while A is the contact surface.
- The thermal exchange coefficient γ can be considered constant (as we shall almost always do) or made dependent on the fluid and motion conditions, typically with relationships involving the Reynolds (for forced convection) or Grashof (for natural one), Nusselt and Prandtl numbers.

- A common, somehow intermediate refinement is to have γ just depend on the fluid velocity tangent to the wall.
- To this end, taking a reference heat exchange coefficient value γ_0 as corresponding to a reference velocity u_0, a widely used relationship is

$$\gamma(t) = \gamma_0 \left(\frac{u(t)}{u_0} \right)^{0.8}$$

where $u(t)$ is the fluid velocity.

- At a simplified level, the radiative heat transfer from a body a to a body b depends on the difference of their absolute (Kelvin) temperatures to the fourth power, i.e.,

$$Q_{ab} = K\left(T_a^4 - T_b^4\right).$$

- The radiative heat transfer coefficient K depends on several things, including the bodies' emissivity and their view factors.
- However in this course the only relevant case will be that of solar radiation, that can be very naturally viewed as a prescribed power flux $[W/m^2]$.

- In this course we shall deal essentially with AC power networks.
- The matter is vast, and simplifications are introduced so as to transmit the necessary concept with the minimum complexity sufficient to explain them.
- In detail (and somehow anticipating) we shall assume
  - a single-frequency synchronous network (quite reasonable if frequency is well controlled, and we do not have the time to deal with the connected "stability" problems),
  - all generators described by a constant voltage behind their internal reactance,
  - linear behaviour of transmission lines,
  - no transformers (we only spend some words on reactive power control) as doing so significantly reduces computations,
  - a one-phase (or equivalently, a *balanced* multiphase) system.
- We shall thus adopt a *phasor-based* modelling approach.

- Any quantity varying (co)sinusoidally with constant frequency ω can be represented as
$$A\cos(\omega t+\theta)=\Re\left(Ae^{j(\omega t+\theta)}\right)=\Re\left(Ae^{j\theta}\,e^{j\omega t}\right)$$
where j is the imaginary unit, $e^{j\omega t}$ yields time dependence, and the phasor $Ae^{j\theta}$ magnitude and phase with respect to a convenient reference.
- This allows for a phasor arithmetic to handle AC networks with frequency "hidden"...
- ...in the term $e^{j\omega t}$ and in the value of frequency-dependent impedances.
- Recall (for the last time) that phasor analysis is for synchronous networks with constant frequency (whence its sometimes encountered alternative name of "static analysis").

Electric networks

Basic equations (essentially to agree notation...)

- Ohm's law $\underline{V} = \underline{Z}\,\underline{I}$ or $\underline{I} = \underline{Y}\,\underline{V}$, where $\underline{V}, \underline{I}$ are voltage and current (phasors) and $\underline{Z}, \underline{Y}$ the complex impedance and admittance, respectively (underline indicates complex numbers). We shall typically express $\underline{Z}$ as $R + jX$ $R, X \geq 0$, and $\underline{Y}$ as $G - jB$ ($G \geq 0$, and mind the minus to have $B \geq 0$), where R, X, G, B are respectively called resistance [Ω], reactance [Ω], conductance [S], and susceptance [S].
- Kirchoff's laws (nothing to say here).
- Power (* denotes the complex conjugate):

complex	$\underline{S} = \underline{V}_{RMS}\underline{I}^*_{RMS} = P + jQ = Ae^{j\phi}$	
apparent	$A = \lvert\underline{S}\rvert = V_{RMS}I_{RMS} = V_{max}I_{max}/2$	[VA],
active	$P = \Re(\underline{S}) = V_{RMS}I_{RMS}\cos\phi$	[W],
reactive	$Q = \Im(\underline{S}) = V_{RMS}I_{RMS}\sin\phi$	[VAR],
	$\cos\phi$	power factor.

Recall that for any sinusoidal signal F, $F_{RMS} = F_{max}/\sqrt{2}$.

Energy equations for rotating masses

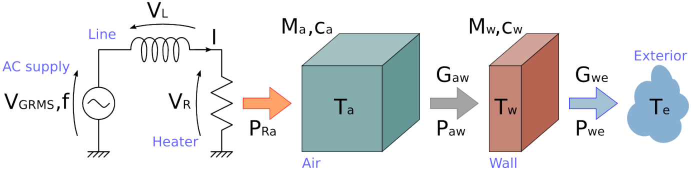
- Many electric generators contain rotating masses, like turbine and alternator rotors.
- Their angular velocity affects the generated frequency (to be controlled).
- The energy equation states that the time derivative of the kinetic energy equals the algebraic sum of powers, i.e.,

$$\frac{d}{dt}\left(\frac{1}{2}J\omega_r^2\right) = P_m - P_e$$

where J is the inertia, ω_r the angular velocity, P_m the mechanical power applied to the shaft (positive if entering the considered machine) and P_e the *active* electric power (positive if generated).

Homework

- Crude simplification of an electrically heated room:



- Data:
  - room dimensions $4\,m \times 4\,m \times 3\,m$ height;
  - only one side wall (30cm thick) exchanges heat, the others are adiabatic;
  - no openings, external temperature $5\,^\circ C$;
  - no heater losses (all of its power is released to air);
  - air density $1.1\,kg/m^3$, specific heat $1020\,J/kg^\circ C$;
  - wall density $2000\,kg/m^3$, specific heat $800\,J/kg^\circ C$;
  - air–wall heat exchange coefficient $10\,W/m^{2\circ}C$;
  - wall–exterior heat exchange coefficient $4\,W/m^{2\circ}C$;
  - AC supply voltage $220\,V\,RMS$, $f = 50\,Hz$, $R = 50\,\Omega$, $L = 10\,mH$.

Homework 01

Assignments
(expected workload: 1.5–2 hours)

- Advice: *first* operate with symbols, *then* put in numbers.
- Assignment 1
  - Write a dynamic model (M1) of the system using energy balances and electrical fundamental equations (i.e., no phasors);
  - count equations and variables, make sure that M1 is closed.
- Assignment 2
  - Write a dynamic model (M2) of the system using energy balances and phasors;
  - count equations and variables, make sure that M2 is closed;
  - analyse M1 and M2: can they be decomposed in cascaded submodels? Why?
- Assignment 3
  - Write the transfer function from P_{Ra} to T_a (linearise if needed);
  - put in numbers and figure out the time scale of thermal phenomena;
  - compare with the time scale of electrical phenomena: what do you conclude?

Lecture 3 (2P)

Previous homework solution

Further considerations on modelling principles

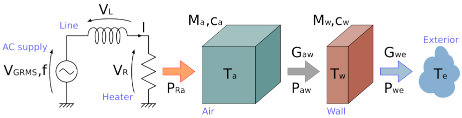
Brief introduction to (wx)Maxima and the Modelica language

Homework

Previous homework solution

(wxMaxima-in-a-nutshell embedded)

Homework 01 - solution



- Model M1 (Masses and thermal conductances are obtained from densities, specific heats and dimensions):

$V_G(t) = V_{GRMS}\sqrt{2}\sin(2\pi f t)$	Source generator
$V_R(t) = RI(t)$	Heater resistor
$V_L(t) = LdI(t)/dt$	Line inductor
$V_R(t) + V_L(t) = V_G(t)$	KVL
$P(t) = V_R(t)I(t)$	Active power
$Q(t) = V_L(t)I(t)$	Reactive power (addendum)
$P_{Ra}(t) = P(t)$	All *active* power into air
$M_a c_a dT_a(t)/dt = P_{Ra}(t) - P_{aw}(t)$	Air energy balance
$M_w c_w dT_w(t)/dt = P_{aw}(t) - P_{we}(t)$	Wall energy balance
$P_{aw}(t) = G_{aw}A_w(T_a(t) - T_w(t))$	Air–wall heat transfer
$P_{we}(t) = G_{we}A_w(T_w(t) - T_e(t))$	Wall–exterior heat transfer
$T_e(t) =$	Exogenous temperature

Note the partition into electric and thermal equations; each has its own boundary conditions, and one such condition is presented by the former to the latter.

Homework 01 - solution

- Equations/variables balance for M1
  - The model has overall 12 equations, 3 of which are differential,
  - its variables (states in red) are
    V_G, V_L, V_R, I, P, Q, P_{Ra}, P_{aw}, P_{we}, T_e, T_a, T_w,
    thus a set of 12, 3 of which are states,
    and there is no algebraic constraint among the states.

  $\Rightarrow$ M1 is a correctly closed ODE model.
- Possible cascade partition for M1
  - Electric equations influence thermal ones via $P_{Ra} = P$
    while the reverse does not hold true, as no *state* electric equation contains any thermal variable.

  $\Rightarrow$ M1 can be viewed as the cascade of
    - an electric subsystem M1e, which is nonlinear owing to $P = V_R I$, with output P_{Ra} and for which we could consider V_G as an input,
    - and a thermal one M1t with input P_{Ra} and output e.g. T_a, which conversely is linear.

Homework 01 - solution

- Model M2: first let us compute P with Maxima by issuing the commands

```
IRMS : VGRMS/(R+%i*w*L);
S    : VGRMS*conjugate(IRMS);
P    : realpart(S);
```

  where obviously $\omega = 2\pi f$.
- This yields

$$P = \frac{RV_{GRMS}^2}{R^2 + (\omega L)^2},$$

- and therefore M2 is

$P(t) = RV_{GRMS}^2/(R^2 + (\omega L)^2)$	Active power
$M_a c_a \dot{T}_a(t) = P_{Ra}(t) - P_{aw}(t)$	Air energy balance
$M_w c_w \dot{T}_w(t) = P_{aw}(t) - P_{we}(t)$	Wall energy balance
$P_{Ra}(t) = P(t)$	All *active* power into air
$P_{aw}(t) = G_{aw} A_w (T_a(t) - T_w(t))$	Air–wall heat transfer
$P_{we}(t) = G_{we} A_w (T_w(t) - T_e(t))$	Wall–exterior heat transfer
$T_e(t)$	Exogenous temperature

Note: from now on we shall often use a dot to indicate time derivatives.

Homework 01 - solution

- Equations/variables balance for M2
  - The model has overall 7 equations, 2 of which are differential,
  - its variables (states in red) are
    P, P_{Ra}, P_{aw}, P_{we}, T_e, T_a, T_w,
    thus a set of 7, 2 of which are states,
    and there is no algebraic constraint among the states.
  - $\Rightarrow$ also M2 is a correctly closed ODE model.
- Possible cascade partition for M2
  - Electric equations were reduced to a boundary condition on P.
  - $\Rightarrow$ M2 can be viewed as a thermal model M2t only,
    with no input and output T_a.
  - We might as well consider M2t to have P_{Ra} as input,
    and in this case the $P_{Ra} \to T_a$ dynamic relationship is linear.

- Let us take some minutes to learn about some of the Maxima commands that we shall use most frequently, i.e.,
  - `matrix`, `invert`, `ident`, `.` and `*`,
  - `ratsimp`, `expand`,
  - `rhs`, `coeff`,
  - and `solve`.

- Compute the transfer function from P_{Ra} to T_a:

```
Paw       : Gaw*(Ta-Tw);
Pwe       : Gwe*(Tw-Te);
se1       : Ma*ca*Tadot = PRa-Paw;
se2       : Mw*cw*Twdot = Paw-Pwe;
solxdot   : ratsimp(solve([se1,se2],[Tadot,Twdot]));
solTadot  : expand(rhs(solxdot[1][1]));
solTwdot  : expand(rhs(solxdot[1][2]));
A         : ratsimp(matrix([coeff(solTadot,Ta),coeff(solTadot,Tw)],
                           [coeff(solTwdot,Ta),coeff(solTwdot,Tw)]));
B         : ratsimp(matrix([coeff(solTadot,PRa),coeff(solTadot,Te)],
                           [coeff(solTwdot,PRa),coeff(solTwdot,Te)]));
Tmat      : ratsimp(invert(s*ident(2)-A).B);
PRa2Ta    : Tmat[1,1];
```

- Result:

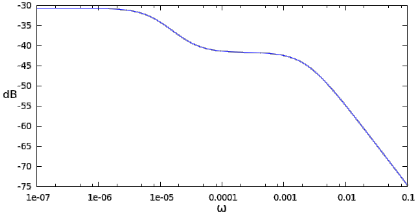
$$G(s) := \frac{T_a(s)}{P_{Ra}(s)} = \frac{c_w M_w s + G_{we} + G_{aw}}{c_a c_w M_a M_w s^2 + (c_w G_{aw} M_w + c_a(G_{we} + G_{aw})M_a)s + G_{aw} G_{we}}$$

Homework 01 - solution

- Put numbers in and plot the Bode magnitude diagram (some more Maxima commands...):

```
load("bode");
Gnum(s)  := ev(subst([Ma=4*4*3*1.1,ca=1020,Gaw=4*3*10,
                      Mw=4*3*0.3*2000,cw=800,Gwe=4*3*4],PRa2Ta));
bode_gain(Gnum(s),[w,1e-7,1e-1]);
```

- Result:



Two time scales:

- one ("fast") relative to the energy storage in the air – order of magnitude, 1000 s,
- the other ("slow") relative to wall storage – order of magnitude, some 10^5 s, i.e., some days.

- Hence, electric phenomena (time scale dominated by $L/R = 0.2\,ms$) are about 10^5 times faster than the faster thermal ones, which are in turn about 10^4 times faster than the slower thermal ones.

Modelica in a nutshell

(more on the matter later on)

- A "hello world" model (`der` is the *time* derivative operator):

```
model HelloWorld
  parameter Real a = -0.2;
  parameter Real b = 1;
  parameter Real c = 1;
  Real x(start=0),u,y;
equation
  der(x)   = a*x+b*u;
  5*y-c*x = 0;
  u        = if time<1 then 0 else 1;
end HelloWorld;
```

- A couple of OMNotebook commands:

```
simulate(HelloWorld,stopTime=20)
plot({u,x,y})
```

- Let us see the result.

Back to homework 01

Modelica realisation of M1 and M2

```
model M1
  parameter Real w=6.28*50;
  parameter Real R=50;
  parameter Real L=0.01;
  parameter Real VGRMS=220;
  Real VG,VL,VR;
  Real I(start=0),P,Q;
  parameter Real Ma=4*4*3*1.1;
  parameter Real ca=1020;
  parameter Real Gaw=4*3*10;
  parameter Real Mw=4*3*0.3*2000;
  parameter Real cw=800;
  parameter Real Gwe=4*3*4;
  Real PRa,Paw,Pwe,Te;
  Real Ta(start=10);
  Real Tw(start=10);
equation
  VG            = VGRMS*sqrt(2)*sin(w*time);
  VG            = VL+VR;
  VR            = R*I;
  VL            = L*der(I);
  P             = VR*I;
  Q             = VL*I;
  Ma*ca*der(Ta) = PRa-Paw;
  Mw*cw*der(Tw) = Paw-Pwe;
  PRa           = P;
  Paw           = Gaw*(Ta-Tw);
  Pwe           = Gwe*(Tw-Te);
  Te            = 5;
end M1;
```

```
model M2
  parameter Real w=6.28*50;
  parameter Real R=50;
  parameter Real L=0.01;
  parameter Real VGRMS=220;
  parameter Real Ma=4*4*3*1.1;
  parameter Real ca=1020;
  parameter Real Gaw=4*3*10;
  parameter Real Mw=4*3*0.3*2000;
  parameter Real cw=800;
  parameter Real Gwe=4*3*4;
  Real P,PRa,Paw,Pwe,Te;
  Real Ta(start=10);
  Real Tw(start=10);
equation
  P             = R*VGRMS^2/(R^2+w^2*L^2);
  Ma*ca*der(Ta) = PRa-Paw;
  Mw*cw*der(Tw) = Paw-Pwe;
  PRa           = P;
  Paw           = Gaw*(Ta-Tw);
  Pwe           = Gwe*(Tw-Te);
  Te            = 5;
end M2;
```

Homework 02

Assignments

(expected workload: 0.5 hours for assignments 1 to 3, less predictable for assignment 4)

- Assignment 1
  - Create an OMNotebook file with M1 and M2 (copy & paste from slides).
- Assignment 2
  - Simulate M1 and M2 for 10 s and verify that the temperature outcomes are consistent;
  - Have a look at the simulation times: what can you observe?
- Assignment 3
  - Suppose that your focus is on the heater behaviour or on the temperature control problem: which model would you prefer for which task? Why?
- Assignment 4
  - Install wxMaxima and OpenModelica (we shall use the OpenModelica Notebook) and familiarise with them.
- From now on, when we have classroom practice bring your laptop.

Lecture 4 (2L)

Previous homework solution

Model structuring
(first-principle models, block- and object-oriented)

The concept of "electric equivalent" in a view to modularisation

Homework

Previous homework solution

Homework 02 - solution

- Let us perform the required simulations and look at the results.
- Considerations:
  - M1 and M2 produce the same temperatures
  - but M1, that also describes fast electric phenomena, is much more computation-intensive;
  - thus suited for studying the heater but not temperature control, while for M2 the reverse is true.
- In other words,
  - in M1 the heater is described as it ought to be for a *component-level* study
  - while in M2 the same component is modelled in a way suitable for *system-level* studies.
- Consequence:
  - in large-scale, multi physics, multi-level systems like energy ones are
    we shall quite often need to model the same component (think of this homework's heater) at different detail levels
    while preserving *interchangeability* of models for the same component.
- Having this in mind, let us proceed.

Model structuring

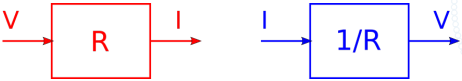
(first-principle models, block- and object-oriented)

- We stick to 1st-principle models (based on dynamic balances) as we often need to analyse/simulate/optimise something that does not yet exist (thus, no data to identify e.g. black- or grey-box models).
- Note: this does *not* mean that identification and estimation never come into play (think e.g. of adaptive control), rather just that the matter does not fit in this course (except for a few words later on).
- We need however to distinguish between block- and object-oriented models.
    - Block-oriented models
        - are *oriented* or *causal*, thus written having their boundary conditions in mind,
        - and connect to one another via *inputs* and *outputs*.
    - Object-oriented models
        - are *a-causal*, thus written independently of their boundary conditions,
        - and connect to one another via *ports*.
- Let us go through an introductory example, and then generalise.

BO (causal) and OO (a-causal) models

Introductory example

- We need to model a resistor $\Rightarrow$ Ohm's law.
- We consider two cases:
  1. the resistor is connected to a fixed voltage generator E, leading with obvious notation to the model
     $V = E$ Voltage generator (boundary condition for the resistor)
     $I = V/R$ Resistor
  2. the resistor is connected to a fixed current generator A, leading this time to
     $I = A$ Current generator (boundary condition for the resistor)
     $V = RI$ Resistor
- Same component, different boundary conditions, different models,
- both of course oriented: in the former case for the resistor V is an input and I an output, in the latter *vice versa*; the two BO resistor models are here respectively



with *input* and *output* connectors.

BO (causal) and OO (a-causal) models

Introductory example

- Now we take a different approach:
  - we identify *ports*, i.e., physical terminals characterising the interface exposed by the modelled component to the outside, in this case the resistor's two pins (a and b to name them) each of which carries a voltage V and a current I, and we write the component's *constitutive equations*, that with obvious notation read

$$RES : \begin{cases} a.I + b.I & = & 0 \\ a.V - b.V & = & R\, a.I \end{cases}$$

    where current is taken positive if entering the pin.
  - We can also write the constitutive equations for the voltage and the current generators, obtaining (mind the current sign convention)

$$VGEN : \begin{cases} a.I + b.I & = & 0 \\ a.V - b.V & = & E \end{cases} \qquad CGEN : \begin{cases} a.I + b.I & = & 0 \\ a.I & = & -A \end{cases}.$$

    .
  - We can finally introduce a ground (with a single pin a), i.e.,

$$GND : a.V = 0.$$

BO (causal) and OO (a-causal) models

Introductory example

- Doing so, the two addressed cases only differ for the generator equations:

Constitutive equations

$RES.a.I + RES.b.I$	$=$	0	Res				
$RES.a.V - RES.b.V$	$=$	$RES.R\ RES.a.I$					
$GEN.a.I + GEN.b.I$	$=$	0	V	$GEN.a.I + GEN.b.I$	$=$	0	C
$GEN.a.V - GEN.b.V$	$=$	$GEN.E$	gen	$GEN.a.I$	$=$	$-GEN.A$	gen
$GND.a.V$	$=$	0	Gnd				

Connection equations

$GEN.a.V$	$=$	$RES.a.V$	pins GEN.a
$GEN.a.I + RES.a.I$	$=$	0	and RES.a
$GEN.b.V$	$=$	$GND.a.V$	pins GEN.b,
$RES.b.V$	$=$	$GND.a.V$	RES.b and
$GEN.b.I + RES.b.I + GND.a.I$	$=$	0	GND.a

- That is, component models are written independently of their connection, they are neither oriented nor closed, communicate via *ports*, and the overall model is closed (thereby determining orientation) by joining the (component-specific) *constitutive equations* and the (port-specific) *connection equations*.

- Ports carry variables, that can be
  - *effort* variables (defined as difference with respect to a reference, think of voltage or temperature), or
  - *flow* variables (defined as flowing through a boundary, think of current or power).
- Connecting N (two or more) ports generates
  - $N-1$ equations per effort variable, stating that it is equal on all connected ports, and
  - one equation per flow variable, stating that its sum over all connected ports is zero.
- Note; all the above has a direct counterpart in OO modelling languages such as Modelica (more on this in due course).

BO and OO models

Distinctive features

- Both types of models allow to *encapsulate* the model's behaviour with respect to its interface, allowing e.g. for scalable detail.
- BO models require the system to be oriented $\Rightarrow$ suitable for control components (block diagram elements) and *complete* controlled systems' models, i.e., for "plants completely built, with control signals and controlled variables already specified as inputs and outputs.
- OO models do not require the system to be oriented $\Rightarrow$ suitable for individual plant components.

- Quite intuitively, we shall use a combination of the two model types.
- Let us exemplify the concepts above, using the Modelica syntax.

BO and OO models

Example on behaviour encapsulation and scalable complexity

- Preliminary definitions: input and output connectors for real signals, electric pin, thermal port.

```
connector signalIn
  input Real s; // Oriented (causal) connection, input
end signalIn;
connector signalOut
  output Real s; // Oriented (causal) connection, output
end signalOut;
connector pin
       Real v;  // Voltage (effort variable)
  flow Real i;  // Current (flow variable)
end pin;
connector thermalPort
       Real T;  // Temperature (effort variable)
  flow Real Q;  // Power (flow variable)
end thermalPort;
```

- Note: the Modelica Standard Library (MSL) already contains most of what we are writing, plus physical types such as “Voltage”, “Current”, “SpecificHeatCapacity”, and so forth: for now we code from scratch to learn the principles and just use “Real” to save time.

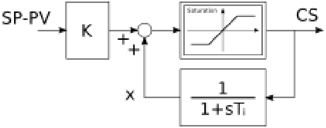
BO and OO models

Example on behaviour encapsulation and scalable complexity

- PI controller: linear model

```
model PI_linear
  signalIn  SP,PV;            // Set Point and Process Variable
  signalOut CS;               // Control Signal
  parameter Real K     = 1;   // Gain
  parameter Real Ti    = 10;  // Integral time
  Real      up,ui(start=0);
equation
  up      = K*(SP.s-PV.s);    // P control
  der(ui) = K/Ti*(SP.s-PV.s); // I control
  CS.s    = up+ui;
end PI_linear;
```

- PI controller: model including saturations and antiwindup




```
model PI_antiwindup
  signalIn  SP,PV;             // Set Point and Process Variable
  signalOut CS;                // Control Signal
  parameter Real K     = 1;    // Gain
  parameter Real Ti    = 10;   // Integral time
  parameter Real CSmin = 0;    // Minimum CS
  parameter Real CSmax = 1;    // Maximum CS
  Real      x(start=0);
equation
  CS.s        = max(CSmin,min(CSmax,x+K*(SP.s-PV.s)));
  x+Ti*der(x) = CS.s;
end PI_antiwindup;
```

- Same interface, different behaviours.

- Capacitor: ideal model

```
model Capacitor_ideal
  pin       a,b;
  parameter Real  C      = 1e-6;  // Capacitance
  parameter Real  Vstart = 0;     // Initial voltage
  Real      V(start=Vstart);
equation
  a.i + b.i = 0;
  a.v - b.v = V;
  a.i       = C*der(V);
end Capacitor_ideal;
```

BO and OO models

Example on behaviour encapsulation and scalable complexity

- Capacitor: model with loss

```
model Capacitor_lossy
  pin       a,b;
  parameter Real  C      = 1e-6;  // Capacitance
  parameter Real  Gloss  = 1e-9;  // Loss conductance
  parameter Real  Vstart = 0;     // Initial voltage
  Real      V(start=Vstart);
equation
  a.i + b.i = 0;
  a.v - b.v = V;
  a.i       = C*der(V)+Gloss*V;
end Capacitor_lossy;
```

- Again, same interface and behaviours of different complexity.

The concept of "electric equivalent" in a view to modularisation

Electric equivalents

- In OO models, ports are naturally associated with energy transfer.
- To understand, consider the typical port with one effort and one flow variable:
- most such couples are keen to be related to such a transfer. Examples are
  - electric, obviously (voltage v, current i) $\Rightarrow$ vi = power;
  - mechanic, translational (position x, force f) $\Rightarrow$ $\dot{x}f$ = power, note the derivative;
  - mechanic, translational (angle φ, torque τ) $\Rightarrow$ $\dot{\varphi}\tau$ = power, note again the derivative;
  - thermal, conductive and convective (temperature T, power Q): note that here power is *not* the product of the two.
- This allows to create *electric equivalent* models, particularly when the flow variable is *linearly* related to a difference of the effort one.

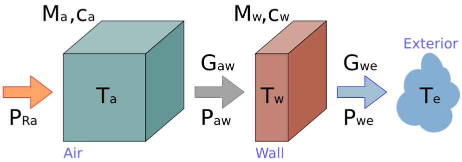
Electric equivalents

- Electric equivalents are thus suited to describe energy-related phenomena when
  - energy storage is related to an effort variable, as in $E = CT$ where C is a *thermal capacity*, and
  - energy transfer is linearly related to differences of that variable, as in $Q = G\Delta T$ where G is a *thermal conductance*.
- They are not (so) suited, therefore, when this is not (so) true, for example
  - for (incompressible) hydraulics, as the mass flowrate is most often related to the square root of a pressure difference, unless dealing with linearised models, or
  - when energy transfer occurs via mass transfer, as in that case the direction of said transfer depends on the sign of some flowrate, i.e., model causality is dictated by the sign of some variables.
- Nonetheless they are a powerful tool, which we shall sometimes employ.

Homework

Homework 03

Assignments
(expected workload: 0.5 hours)



- Assignment 1
  - Create an electric equivalent model for the thermal part of the system used for homework 01, shown above for convenience.
- Assignment 2
  - Classify components into "source" or "boundary condition", "storage", and '"flow" or "transfer" ones, motivating your statements.

Lecture 5 (2L)

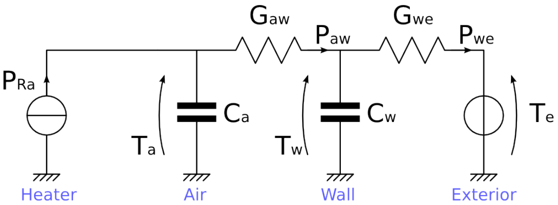
Previous homework solution

Models and control problems

Models of electric components for power and frequency control: generators

Previous homework solution

- Required model:



where $C_a = M_a c_a$, $C_w = M_w c_w$.

- Component classification:

Current generator	heater	boundary condition
Voltage generator	exterior	boundary condition
Conductors	air–wall and wall–exterior convection	flow
Capacitors	air and wall	storage

- Note: we prefer "boundary condition" to "source" (or "sink") because the latter term(s) also refer to the *sign* of some flow variable, which is not relevant to model *structuring*.

Models and control problems
(electric networks)

- In the control of electric networks, two types of problems are encountered:
  - **power and cost control**, i.e.,
    deliver the required power to all utilisers while
    - minimising costs (and possibly emissions) *globally* (general interest),
    - or maximising sold power at the minimum cost *for one or more generators* (particular interest),

    which are often conflicting objectives;
  - **energy quality control**, i.e.,
    deliver electricity at the required voltage and frequency, which necessarily involves cooperation as all the generators (and loads) are coupled by the network (no particular interest makes sense here).
- The above problems are apparently intertwined, in a way that however depends on the generator operation.
- Also, only generators act to provide control: the power demand by loads is considered exogenous (i.e., a disturbance).

Models and control problems

- In this respect, two generator types are in fact distinguished:
  - with rotating masses, i.e., with an alternator (e.g. thermo, nuclear, hydro, wind, tidal, thermal – or "thermodynamic" – solar),
  - and without rotating masses, i.e., with an inverter (e.g. photovoltaic solar).
- Rotating masses inherently couple power and frequency, as a power excess or deficiency accelerates or decelerates the masses, thereby altering frequency.
- This does not hold true when no rotating mass is present.

- Other problems exist that do not fit in the space of this course, like e.g. reactive power control.
- Since we aim at system-level principles and models, we shall examine generators having the thermoelectric case as reference, and then generalise to a feasible extent.
- Our KPIs will be power and frequency error, generation cost, and transmission losses.

Models and control problems

- When moving from generation to network control, another relevant problem is encountered:
    - **load flow**, i.e.,
      delivering power without overloading transmission lines, and possibly minimising line losses.
- Load flow can just provide constraints for power and quality control to avoid overloads (and we shall just say some words on this)
- or be part of the overall optimisation, leading to the *optimal flow* problem that we cannot treat here.

- Finally, as we shall see soon, power and quality problems require to abstract different generator and network element interfaces.
- Let us now proceed to model generators, starting with the problem of (active) power – and frequency – control.
- As anticipated, we start from the thermoelectric case and then extend the ideas.

Models of electric components

for power and frequency control

Generators

- These generators include a rotating mass, whence the power–frequency coupling.
- We consider as reference case an *islanded* generator (no other generators, just one feeding an equivalent network load).
- Synthesis of operation:
  - fuel burns in a *furnace* and produces heat;
  - heat turns water into superheated steam;
  - steam moves a turbine;
  - the turbine moves the alternator,
- The generator has its internal controls, which we do not represent,
- and we also disregard the details about the water/steam path (circulation, once-through)...
- since the matter is treated in dedicated courses, and is not to be represented in our system-level models.

- We start from the *prime mover*, i.e., the system having fuel as inlet and mechanical power for the alternator as outlet.
- For simplicity we take as exogenous input the combustion power P_c released to the *main energy storage* (steam); fuel consumption will come into play later on.
- In so doing we neglect heat storage in the combustion chamber and the flue gas path as they are very small w.r.t. those in the metal and water/steam path, which we assume thermally coherent.
- The stored energy balance is thus

$$\dot{E} = P_c - P_{loss} - P_t$$

where P_{loss} is the power lost to the external environment and P_t the power drawn by the turbine.

- We simplistically assume that the main energy storage is composed of saturated steam and its mass is constant; we thus relate P_{loss} to the difference between the saturation temperature at the steam pressure p (which thereby comes to represent the stored energy) and the external temperature, as

$$P_{loss} = G_{loss}(T_{sat}(p) - T_{ext})$$

  where G_{loss} is an equivalent thermal conductance.
- Even more simplistically, thus, since in general $T_{sat}(p) \gg T_{ext}$, we write

$$P_{loss} = K_{loss}E/M$$

  where K_{loss} is a convenient parameter and the division by M represents the fact that P_{loss} depends on the steam *specific state*; K_{loss} is of course related to the dispersing surface.

- We now make another simplistic assumption by disregarding the superheating that steam undergoes prior to traversing the turbine valve, and assume P_t to depend on the steam pressure (that is, on E/M) and the turbine valve opening $\theta \in [0,1]$ as

$$P_t = \theta K_{draw} E/M$$

  where K_{draw} is another parameter.

- The mechanical power reaching the alternator is then obtained by accounting for a mechanical efficiency, assumed constant and denoted by η_m, as

$$P_m = \eta_m P_t.$$

- Putting it all together, we have

$$
\begin{cases}
\dot{E} = P_c - K_{loss}E/M - \theta K_{draw}E/M \\
P_m = \eta_m \theta K_{draw}E/M
\end{cases}
$$

- Note that $[K_{loss}E/M] = [K_{draw}E/M] = [W]$ since θ and η_m are adimensional, thus $[K_{loss}/M] = [K_{draw}/M] = [1/s]$ and we can write

$$
\begin{cases}
\dot{E} = P_c - \left(\dfrac{1}{T_{loss}} + \dfrac{\theta}{T_{draw}} \right) E \\
P_m = \dfrac{\eta_m}{T_{draw}} E\theta
\end{cases}
$$

- In the state equation above, T_{loss} and T_{draw} are interpreted as the time constants with which energy is respectively lost into the environment and yielded to the alternator at full throttling (turbine) valve opening.

- Notice that the plant size, intuitively indicated e.g. by (some nominal value for) the contained water/steam mass M, in this formulation comes to be represented by the introduced time constants (larger plant, larger M, larger T_{loss} and T_{draw}).

- Of course models like this one are in the large *extremely* coarse, and for real-life uses they could only have local validity around an operating point.
- Treating such aspects strays from the course, however, and therefore we shall just use the model *as is* since this is more than enough for our purposes.

- We can furthermore introduce a generator nominal power P_n and denote by T_{rest} the time required to restore the generator to its "nominal energy storage", thereby defined as $E_n = P_n T_{rest}$. Dividing equations by E_n hence gives

$$\begin{cases} \dot{e} & = & \dfrac{1}{T_{rest}} p_c - \left(\dfrac{1}{T_{loss}} + \dfrac{\theta}{T_{draw}} \right) e \\ p_m & = & \eta_m \dfrac{T_{rest}}{T_{draw}} e\,\theta \end{cases}$$

where $p_c = P_c/P_n$ and $p_m = P_m/P_n$ are respectively the normalised combustion and mechanical powers, while $e = E/E_n$ is the normalised energy storage (*not* the steam specific energy, beware).

- As such, indicating – for this discussion only – the steam specific energy $[J/kg]$ with e_{spec}, we could better rewrite the second model equation (mind the different meanings of e_{spec} and e) as

$$P_m = \eta_m \, K_{draw} e_{spec} \, \theta = \eta_m \, K_{draw} \frac{E}{M} \, \theta = \eta_m \, K_{draw} \frac{P_n \, T_{rest}}{M} e \, \theta$$

- Thus, normalising by P_n,

$$p_m = \eta_m \, K_{draw} \frac{T_{rest}}{M} \, e \, \theta$$

- Overall, the quantity $k_\theta := K_{draw} \, T_{rest} / M$ acts as sort of a "valve gain" cascaded to the mechanical efficiency. Investigating its role would however require relating also the contained *mass* to the energy state, which we do not want to do for our purposes.

- Thus, we shall simply write

$$\begin{cases} \dot{e} & = & \dfrac{1}{T_{rest}} p_c - \left(\dfrac{1}{T_{loss}} + \dfrac{\theta}{T_{draw}} \right) e \\ p_m & = & \eta_m k_\theta \, e \, \theta \end{cases}$$

and for simplicity (without didactic loss for this course) assume $k_\theta = 1$, hence omitting it hereinafter.

- Given all the above, we shall take as generator model the dynamic system

$$\begin{cases} \dot{e} & = & \dfrac{1}{T_{rest}} p_c - \left(\dfrac{1}{T_{loss}} + \dfrac{\theta}{T_{draw}} \right) e \\ p_m & = & \eta_m \, e \, \theta \end{cases}$$

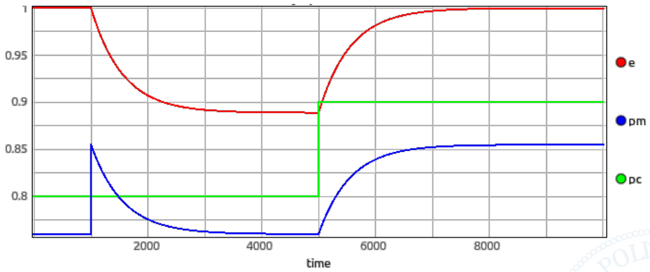
- Now, determine the equilibrium of the previous model for constant inputs $\overline{p}_c, \overline{\theta}$, which produces

$$\overline{e} = \frac{T_{draw} T_{loss} \overline{p}_c}{T_{rest}(T_{draw} + T_{loss}\overline{\theta})}, \quad \overline{p}_m = \eta_m \overline{e} \overline{\theta}.$$

- Let us simulate the prime mover model in Modelica, starting at the equilibrium and applying steps to θ and p_c, with

```
model SimpleThermoElecGenPM
  parameter Real Pn       = 100;
  parameter Real Trest    = 500;
  parameter Real Tdraw    = 500;
  parameter Real Tloss    = 1e9;
  parameter Real etam     = 0.95;
  parameter Real thetabar = 0.8;
  parameter Real pcbar    = 0.8;
  Real e(start=Tdraw*Tloss*pcbar/Trest/(Tdraw+Tloss*thetabar));
  Real pc,pm,theta;
equation
  der(e) = pc/Trest-(1/Tloss+theta/Tdraw)*e;
  pm     = etam*e*theta;
  theta  = if time<1000 then thetabar else thetabar+0.1;
  pc     = if time<5000 then pcbar    else pcbar+0.1;
end SimpleThermoElecGenPM;
```

- Issuing `simulate(SimpleThermoElecGenPM,stopTime=10000)` and then `plot({e,pm,pc})` produces



- The θ step ($t = 1000\,s$) produces a sudden p_m response, then since p_c is constant p_m settles back to the previous value while e decreases and settles, both transients being dominated by the storage time constant.
- The p_c step ($t = 5000\,s$) makes both p_m and e increase and settle with the storage time scale.

- The linearised model in the vicinity of the equilibrium, setting $\Delta p_c = p_c - \overline{p}_c$, $\Delta\theta = \theta - \overline{\theta}$ and $\Delta e = e - \overline{e}$, $\Delta p_m = p_m - \overline{p}_m$, and taking as outputs both Δp_m and Δe, is

$$\begin{cases} \Delta\dot{e} = -\left(\dfrac{1}{T_{loss}} + \dfrac{\overline{\theta}}{T_{draw}}\right)\Delta e + \begin{bmatrix} -\dfrac{\overline{p}_c T_{loss}}{T_{rest}(T_{draw} + T_{loss}\overline{\theta})} & \dfrac{1}{T_{rest}} \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta p_c \end{bmatrix} \\ \begin{bmatrix} \Delta p_m \\ \Delta e \end{bmatrix} = \begin{bmatrix} \eta_m \overline{\theta} \\ 1 \end{bmatrix} \Delta e + \begin{bmatrix} \dfrac{\eta_m \overline{p}_c T_{draw} T_{loss}}{T_{rest}(T_{draw} + T_{loss}\overline{\theta})} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta p_c \end{bmatrix} \end{cases}$$

- Notice that it is asymtptocally stable as T_{draw}, T_{loss} and $\overline{\theta}$ are all positive.

- The corresponding transfer matrix is

$$
\Gamma(s) = \begin{bmatrix} \Gamma_{\theta m}(s) & \Gamma_{cm}(s) \\ \Gamma_{\theta e}(s) & \Gamma_{ce}(s) \end{bmatrix} = \begin{bmatrix} \dfrac{\Delta p_m(s)}{\Delta\theta(s)} & \dfrac{\Delta p_m(s)}{\Delta p_c(s)} \\ \dfrac{\Delta e(s)}{\Delta\theta(s)} & \dfrac{\Delta e(s)}{\Delta p_c(s)} \end{bmatrix} = \dots
$$

$$
= \frac{1}{1+s\frac{T_{draw}T_{loss}}{T_{draw}+T_{loss}\overline{\theta}}} \begin{bmatrix} \frac{\eta_m T_{draw}^2 T_{loss}\overline{p}_c}{T_{rest}(T_{draw}+T_{loss}\overline{\theta})}(1+sT_{loss}) & \frac{\eta_m T_{draw} T_{loss}\overline{\theta}}{T_{rest}} \\ -\frac{T_{draw} T_{loss}^2 \overline{p}_c}{T_{rest}(T_{draw}+T_{loss}\overline{\theta})} & \frac{T_{draw}T_{loss}}{T_{rest}} \end{bmatrix}
$$

- Note that all the elements have relative degree 1 except for $\Delta p_m/\Delta\theta$, which has relative degree 0; this is consistent with the simulated responses.

- The Maxima script used for the previous computations is reported for reference:

```
/* Model
*/
se    : pc/Trest-(1/Tloss+theta/Tdraw)*e;
pm    : etam*e*theta;

/* Equilibrium
*/
ebar  : rhs(solve(subst([pc=pcbar,theta=thetabar],se),e)[1]);
pmbar : ratsimp(etam*ebar*thetabar);

/* Linearised model state space matrices
*/
A     : subst([pc=pcbar,theta=thetabar,e=ebar],jacobian([se],  [e]));
B     : subst([pc=pcbar,theta=thetabar,e=ebar],jacobian([se],  [theta,pc]));
C     : subst([pc=pcbar,theta=thetabar,e=ebar],jacobian([pm,e],[e]));
D     : subst([pc=pcbar,theta=thetabar,e=ebar],jacobian([pm,e],[theta,pc]));

/* Linearised model transfer matrix. Note that here A is actually scalar; the
   syntax for the matrix-A case would be C.invert(s*ident(size_of_a)-A).B+D
*/
Gamma : factor(C.B*invert(s-A)+D);
```

- For simplicity we identify here cost and fuel consumption (i.e., we do not include here plant maintenance, personnel and so on).
- Combustion is not equally efficient at all *plant loads*, i.e. – looking at the *thermal* load – for all values of p_c.
- A *specific* consumption c_s ([kg of fuel per J], i.e., [kg/s of fuel per W]) is thus defined, which is typically a decreasing function of P_c in the admissible operation range, which normalised as done for p_c, in turn corresponds to an interval $(p_{c,min}, p_{c,max})$:
  - $p_{c,min}$ is the minimum "technical" load below which the generator cannot be operated, and may be something like 0.2-0.25,
  - while $p_{c,max}$ is the maximum "guaranteed" power for the generator to work safely, and can be slightly greater than the unity, say 1.05–1.10, to allow transient "exceptional" power releases to the network.
- Given the above, the fuel mass flowrate w_c and p_c are related by

$$w_c = c_s(p_c P_n) p_c P_n,$$

which in our model we use to compute w_c while maintaining p_c as control input, for simplicity and consistency with our scope.

- The active power demanded by the load is exogenous for the generator; denote it with P_e.
- Thus, the energy equation for the rotating mass (turbine and alternator) reads

$$J\omega\dot{\omega} = P_m - P_e$$

where J is the total inertia seen at the shaft (have this in mind when we shall talk about multiple generators and not an islanded one as we are doing now), and ω the angular velocity (for us identified with the electric frequency).
- The equation above yields possible equilibria at any ω, provided that the (constant) values $\overline{P}_m$ and $\overline{P}_e$ coincide. Linearising with obvious notation, thus,

$$\Delta\dot{\omega} = -\frac{P_m - P_e}{J\overline{\omega}^2}\Delta\omega + \frac{1}{J\overline{\omega}}(\Delta P_m - \Delta P_e)$$

- Assuming thus that ω is regulated – which we shall guarantee shortly – at its desired value ω_o and recalling that at the equilibrium $P_m = P_e$, we have

$$\dot{\Delta\omega} = \frac{1}{J\omega_o}(\Delta P_m - \Delta P_e)$$

- Normalising $P_{m,e}$ with P_n and ω with ω_o and using δ for the variations of normalised quantities, finally,

$$\frac{\Delta\dot{\omega}}{\omega_o P_n} = \frac{1}{J\omega_o}\left(\frac{\Delta P_m}{\omega_o P_n} - \frac{\Delta P_e}{\omega_o P_n}\right)$$

- which means, rearranging,

$$\delta\dot{\omega} = \frac{P_n}{J\omega_o^2}\left(\delta P_m - \delta P_e\right)$$

  Note that $[P_m/J\omega_o^2] = [W/J] = [1/s]$; the quantity $J\omega_o^2/P_m$ is typically denoted by T_A.

- No homework this time, but please review lecture notes.

Lecture 6 (2L)

Models of electric components for power and frequency control
- Generators (cont'd, and including control)
- Network
- Extending from thermo to other generator types

- We have described the (linearised) $(\Delta\theta, \Delta p_c) \rightarrow (\Delta p_m, \Delta e)$ relationship as a first-order transfer matrix $\Gamma(s)$.
- This is adequate at system level, where in fact slightly more complex models (up to the third order) are used that account more precisely for the generator structure, which here we disregard for simplicity.
- Recall that the energy content is related to the steam pressure p in the generator; in fact dealing with p instead of a generic "energy" content e is a major reason for using the mentioned slightly more complex models.
- For consistency and realism we shall thus talk about *pressure* control, that however will just be mentioned: recall that this means in any case controlling the energy storage in the generator.

- Approaching thus control, our first problem is how to regulate the mechanical power released to the alternator, and the generator pressure.
- This can be done in three ways, named "boiler follows", "turbine follows", and "variable (or sliding) pressure".

- **Boiler follows**
  Idea: use θ to modulate P_m
  and P_c – i.e. w_c – to keep p as constant as possible.
  Pros: very fast response of P_m to its set point.
  Cons: transient pressure variations, generally not excessive but of noticeable entity, and potentially detrimental especially in the long run (stress).
- **Turbine follows**
  Idea: use θ and P_c the other way round w.r.t. boiler follows.
  Pros: far better pressure control.
  Cons: power response much slower.
- **Sliding pressure**
  Idea: set θ to its max value (valves fully open)
  and control P_m by acting on w_c.
  Pros: minimum turbine upset.
  Cons: power response *extremely* slow (the boiler is not even concerned with restoring its pressure).

- As a small but control-relevant refinement, we need to describe actuators, since of course neither θ nor P_c can be varied instantaneously.
- For this purpose, we introduce a diagonal transfer matrix

$$A(s) = \begin{bmatrix} \dfrac{1}{1+sT_\theta} & 0 \\ 0 & \dfrac{1}{1+sT_{P_c}} \end{bmatrix}$$

accounting for the actuators' dynamics, so that

$$\begin{bmatrix} \Delta\theta(s) \\ \Delta P_c(s) \end{bmatrix} = A(s) \begin{bmatrix} \Delta\theta_c(s) \\ \Delta P_{cc}(s) \end{bmatrix}$$

- In the above equation θ_c and P_{cc} are the actuator *commands*, while θ and P_c represent the real actions on the plant. This means that we actually have to deal with the transfer matrix $\Gamma(s)A(s)$.

- To lighten our notation, however, we shall from now on drop the "c" subscript standing for "command" and consider $A(s)$ as part of the process, thereby always dealing with the cascade of A and Γ, but calling its inputs θ and P_c as we did up to now while not accounting for the actuators, i.e., containing just Γ and not A.
- In other words, we shall write

$$\begin{bmatrix} \Delta P_m(s) \\ \Delta e(s) \end{bmatrix} = \Gamma_{ap}(s) \begin{bmatrix} \Delta\theta(s) \\ \Delta P_c(s) \end{bmatrix}$$

where $\Gamma_{ap}(s) = \Gamma(s)A(s)$

- Coming back to the main subject, the policy to use is chosen – among the mentioned three – based on the generator size, its type (e.g. circulating vs. once-through, details in specific courses), role in the network (base load vs. fast dispatching), and cost of transients (overfiring and so on).
- Sometimes the policy is varied over time, depending e.g. on the present load (generated power).

- In any case, once the control variable for P_m is chosen (it can be either θ or P_c, and we denote it by u_P) and given the generator structure, in the linearised context we have to do as the system to be controlled with a transfer function, derived from from $\Gamma_{ap}(s)$ – recall the notation remark just given) – that we shall generically call $G(s)$, i.e.,

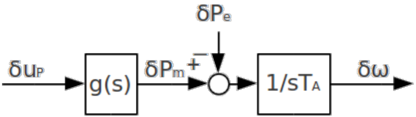
$$G(s) = \left.\frac{\Delta P_m(s)}{\Delta u_P(s)}\right|_{\text{Gen. structure, Pm control policy}}$$

- Of course we can normalise also $G(s)$ with respect to the nominal power P_n and a nominal control u_{Pn}, obtaining

$$g(s) = \frac{\delta P_m(s)}{\delta u_P(s)}$$

where u_{Pn} is 1 or again P_n if u_P is θ or P_c, respectively.

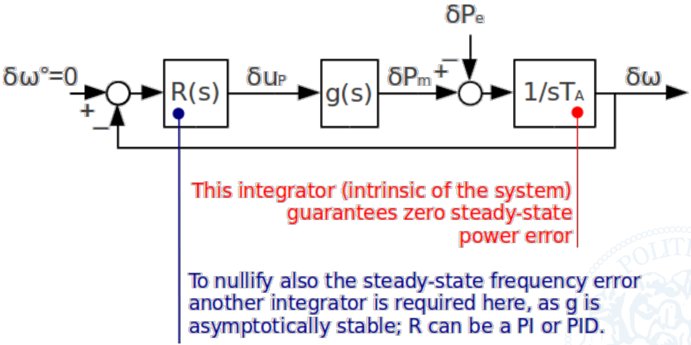
- In any case, the plant to be controlled (an islanded generator, remember) is described as



where the normalised variation δP_e of the power demanded by the load acts as a disturbance.

- Recall that $g(s)$ has low order (here two, with slightly mode refined models say four–five) and is asymptotically stable.

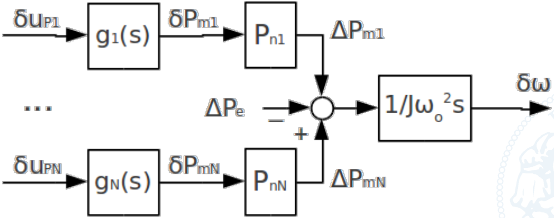
- For the islanded generator case, control can be carried out as shown below:



- This concludes the reference case. Let us now consider multiple generators in a network.

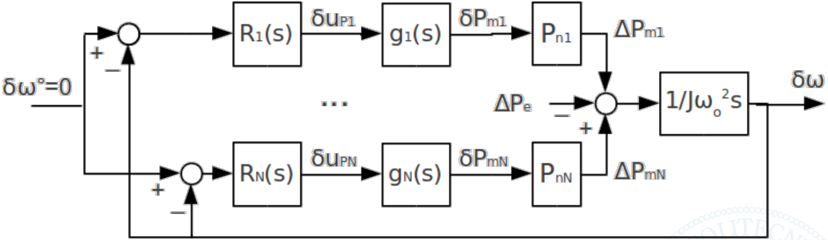
Network

- In the case of multiple generators, at our system level we introduce the *rigid synchronous network* hypothesis: all the masses rotate together at the same speed, no swinging.
- In this case all the mechanical powers (*not the normalised powers*, beware) sum together, while there is still a *single* electric power demand (the total for the network) subtracted from them. The system under control is thus



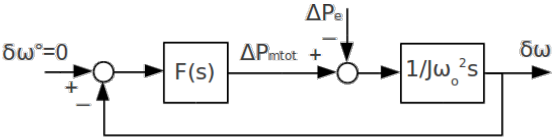
where J is the total network inertia (there is no overall T_A as each of the N generators has its own P_n).

- The scheme for the islanded generator is easily extended to multiple generators as



- Here too the network intrinsic integrator $(1/J\omega_o^2 s)$ guarantees zero steady-state power error.
- However the regulators in this scheme cannot encompass integrators, because in that case the generation distribution would not be controllable.

- To understand why, observe that the scheme is equivalent to



- where ΔP_{mtot} is the total mechanical power variation (*not normalised*), and

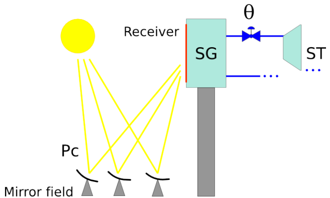
$$F(s) = \sum_{i=1}^{N} R_i(s) g_i(s) P_{ni}.$$

- Possible integrators in the R_i regulators would thus be in parallel, whence the controllability loss.

- Solution: to have zero steady-state frequency error there must be *one* integrator. thus
  - employ for the *primary regulators* $R_i(s)$ a type 0 structure (most frequently a pure proportional term K, whence the name "$K\Delta f$" frequently encountered for them),
  - and introduce a *secondary frequency control* in the form of a single integrator per network, having as input the frequency error;
  - the output of the secondary regulator acts as an additive correction on the output of each $K\Delta f$ controller via a gain β_i that can be different for each generator, and dictates how much that generator will be asked to participate to the secondary control.

Some words on other generator types

- Quite traditional type (receiver = SG)

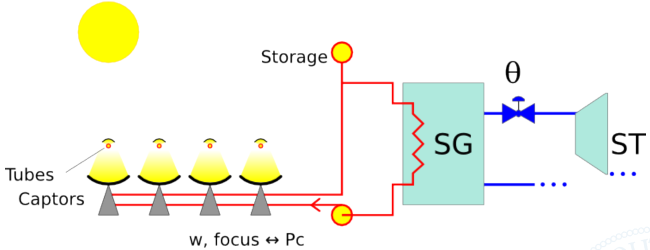


- The primary energy source (sun) is clearly uncontrollable
  ⇒ one can think of an exogenously varying prmary energy source.
- What is controllable, by focusing or de-focusing the mirrors, is the *amount* of the available power that the plant actually draws
  ⇒ the mirror focusing plays more or less the role of P_c, but subject to the variability above. Note that in general mirrors are kept at full focus for efficiency, so the variability *does* matter.
- Main problems: the mentioned variability and the difficulty of introducing "large" energy storages (w.r.t. that provided by the SG alone).

Solar plant

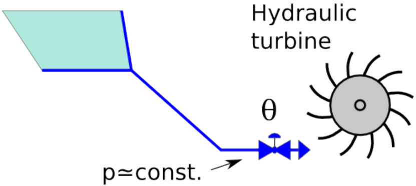
of the thermal (thermodynamic) type

- More recent type (an example)



- The primary fluid (e.g., molten salt) allows for a significant heat storage, thereby smoothing the prime source variability as seen by the SG
  $\Rightarrow$ the situation is more similar to the reference thermo case.
- Note: the figure is *highly* simplified as for the storage management.

- Very simple scheme:



  - At the generator control time scale, the energy reserve (basin) may be considered practically unlimited.
  - The mechanical power can be assumed to be an algebraic function of the turbine valve command, i.e.,

$$P_m = f(\theta),$$

    although rigorously depending also on the rotor and fluid speed (losses are due e.g. to residual jet kinetic energy).

- ...and so forth for other generator types.

Generators without rotating masses

(just a couple of words)

- Notable examples are photovoltaic generators and fuel cells.
- In both cases there is a primary source, either well controllable (fuel) or vastly exogenous (solar radiation).
- Then, there may or may not be a significant reserve (this is not the case e,g, for a photovoltaic generator without batteries).
- The process of generating electricity does not involve mechanics and is mostly solid-state, thus one can think that power can be commanded independently of frequency, while synchronisation with the network is always guaranteed.
- In one word, these generators too fall in our abstract model structure.

- No homework this time too, but (again) review lecture notes as we shall continue with a practice session. Next time bring a pocket calculator and some semilogarithmic paper sheets.

Classroom practice

Applying the OO paradigm

- Up to now we have been using for compound models (e.g., those introduced for control) a BO approach.
- Of course an OO one can be adopted as well.
- To this end, first define a connector representing rigid synchronous coupling (it makes frequency equal in all the connected models, thus same speed, thus no swinging) as

```
connector PowerFreqPort
       Real f; // Frequency
  flow Real P; // Power
end PowerFreqPort;
```

- Then define a network model containing the single inertia, as

```
model ElectricNetworkPF
  signalIn      Pe;              // Electric power (exogenous)
  PowerFreqPort Pg;              // Port to connect all generators
  parameter     Real J  = 1000; // Inertia
  parameter     Real fo = 50;   // Nominal (and initial) frequency
  Real          f(start = fo);  // Frequency
equation
  der(f) = (Pg.P-Pe)/(J*8*3.14^3*fo^2); // der(2*pi*f)=(Pg-Pe)/(J*(2*pi*fo)^2)
  f      = Pg.f;
end ElectricNetworkPF;
```

- Finally, write a (thermo) generator OO model as

```
model SimpleThermoElecGenPF
  signalIn       theta;          // Throttling valve command
  signalIn       Pc;             // Combustion power command
  PowerFreqPort  Pg;             // Port to network
  parameter Real Pn        = 100;
  parameter Real Trest     = 500;
  parameter Real Tdraw     = 500;
  parameter Real Tloss     = 1e9;
  parameter Real etam      = 0.95;
  parameter Real thetabar = 0.8;
  parameter Real pcbar     = 0.8;
  Real e(start=Tdraw*Tloss*pcbar/Trest/(Tdraw+Tloss*thetabar));
  Real pc,pm;
equation
  der(e) = pc/Trest-(1/Tloss+theta/Tdraw)*e;
  pm     = etam*e*theta;
  pc     = Pc/Pn;
  pm     = Pg.P/Pn;
end SimpleThermoElecGenPF;
```

Summing up...

- ...let us do some simulations with an islanded generator managed with BF, TF, SP, with primary only and primary/secondary control, and comment.
- A more complete Modelica library for the concepts just exposed, including control-related ones and using the Modelica Standard Library (MSL) connectors, is available on the course site (`AES2012_PFcontrol.mo`).

Classroom practice

Power and frequency control – primary and secondary regulation

Lessons to learn

i.e., just one slide for this lecture's objectives

- View islanded generator control as a composition of primary and secondary actions behaving like a proportional and an integral one, understanding their roles via the typical transient caused by a step variation in the demanded power.
- Transpose the same idea of "equivalent PI" to the networked case, interpreting primary (distributed) and secondary (centralised) actions along the same reasoning as for the islanded case.
- Discuss the role of secondary distribution coefficients in the networked case.
- Understand the usefulness of a "low-frequency" (from daily to down to say 15-minutes scale) scheduling of the power to be generated and its distribution,...
- ...that is sometimes called *tertiary control*; this is a basically feedforward action aimed at optimising cost, and will be the subject of the next lectures.

Lecture 9 (2L)

Generation cost optimisation

- The basics and an introductory example
- A methodology and some considerations

The basics and an introductory example

- To understand, better *not* to reason with normalised quantities.
- Consider a network with N generators: at any given moment, the total mechanical power must equal the electric power demand, i.e.,

$$\sum_{i=1}^{N} P_{mi} = P_e$$

- Primary and secondary control can ensure this, and also keep frequency to the set point (if the power request is feasible, of course). But what about cost?
- Knowing the efficiency curve of each generator,and talking for better precision of "generated" (subscript g) rather than "mechanical" power, one can naturally write N functions relating each P_{gi} $[W]$ to a "cost rate" c_i $[€/s]$ or $[€/h]$.
- Clearly, $c_i(P_{gi})$ is a monotonically increasing function.
- Of course each generator has limits, i.e., $P_{gi,min} \leq P_{gi} \leq P_{gi,max}$.

- Supposing for simplicity that the purpose is to minimise the overall cost, it can be stated as that of minimising the overall cost rate, hence as

$$
\begin{aligned}
\min \sum_{i=1}^{N} & c_i(P_{gi}) \\
\text{s.t.} \quad & \sum_{i=1}^{N} P_{gi} = P_e, \\
& P_{gi,min} \leq P_{gi} \leq P_{gi,max}, \;\; i = 1 \ldots N.
\end{aligned}
$$

- Note: the problem can be much more complex as *sets* of generators may aim at minimising *their* cost while together generating a desired P_e or share of P_e, but we do not have enough time do delve into such further details. Incidentally, we shal foresee other sources of additional complexity.
- Let us then understand the principles, and defer any possible refinement to specialised courses.

- Most frequently, cost models are polynomial and up to cubic in power. A typical form is

$$c_i(P_{gi}) = (k_{g1}P_{gi} + k_{g2}P_{gi}^2 + k_{g3}P_{gi}^3)k_F + k_{om0} + k_{om1}P_{gi}$$

where

c_i	$[€/h]$	is the cost rate,
P_{gi}	$[W]$	is the generated power,
k_{gj}, $j = 1 \dots 3$	$[J/(hW^j)]$	are cost coefficients for pure generation,
k_F	$[€/J]$	is the fuel cost per unit of *energy*,
$k_{om\ell}$, $\ell = 0, 1$	$[€/(hW^\ell)]$	are cost coefficients for Operation & Maintenance.

- Note that, quite logically, the pure generation cost nullifies for $P_{gi} = 0$, while the O&M cost does not.

- Why cubic?
- Because the fuel to generated power ratio $r_{fP}(P_g) = Q_f(P_g)/P_g$, where Q_f is the power yielded by fuel $[W]$ and P_g the generated power, thus making r_{fP} adimensional, typically has a minimum at the *optimal operating point* (by construction generally close to the maximum or *rated* power $P_{g,max}$).
- A good way to synthetically model this is to describe function $r_{fP}(P_g)$ as a *parabola*, specifying
  - the optimal (minimum) fuel to generated power ratio r_{fP}^o,
  - the fraction p_g^o of $P_{g,max}$ corresponding to that optimal ratio, where p_g is defined as $P_g/P_{g,max}$,
  - and the fuel to generated power ratio r_{fP}^{ml} ($> r_{fP}^o$) at the minimum sustainable load, i.e., at $p_g^{ml} = P_{g,min}/P_{g,max}$,
- which gives

$$r_{fP}(p_g) = r_{fP}^o + \frac{r_{fP}^{ml} - r_{fP}^o}{\left(p_g^o - p_g^{ml}\right)^2}\left(p_g - p_g^o\right)^2$$

thus

$$r_{fP}(P_g) = r_{fP}^o + \frac{r_{fP}^{ml} - r_{fP}^o}{\left(P_g^o - P_{g,min}\right)^2}\left(P_g - P_g^o\right)^2$$

- Alternatively, $r_{fP}(P_g)$ can be obtained by interpolating experimental points (and again, a *parabola* normally suffices).
- Therefore, no matter how $r_{fP}(P_g)$ is obtained, $Q_f(P_g)$ can be expressed as

$$Q_f(P_g) = r_{fP}(P_g)P_g = \left(r_{fP}^o + \frac{r_{fP}^{ml} - r_{fP}^o}{\left(P_g^o - P_{g,min}\right)^2} \left(P_g - P_g^o\right)^2 \right) P_g$$

that apparently contains powers of P_g from one to three, like we just wrote for the term $k_{g1}P_{gi} + k_{g2}P_{gi}^2 + k_{g3}P_{gi}^3$.
- In detail,

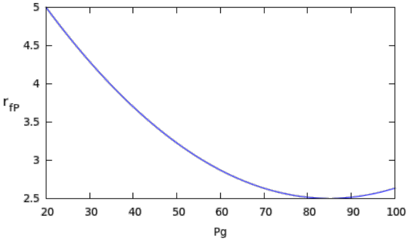
$$k_{g1} = \frac{r_{fP}^{ml} P_g^{o\,2} - r_{fP}^o P_{g,min} \left(2P_g^o - P_{g,min}\right)}{\left(P_g^o - P_{g,min}\right)^2}, \quad k_{g2} = \frac{2P_g^o \left(r_{fP}^o - r_{fP}^{ml}\right)}{\left(P_g^o - P_{g,min}\right)^2},$$

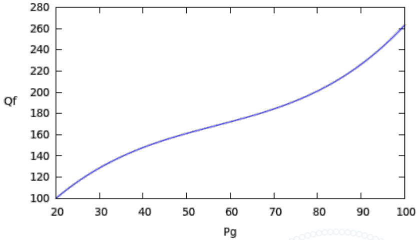
$$k_{g3} = \frac{r_{fP}^{ml} - r_{fP}^o}{\left(P_g^o - P_{g,min}\right)^2}.$$

Problem statement

Generator cost models – an example

- The above cost model with $r_{fP}^{o} = 2.5$, $r_{fP}^{ml} = 5$, $P_g^o = 85$, $P_{g,min} = 20$ (supposing $P_{g,max} = 100$ for the plots) produces





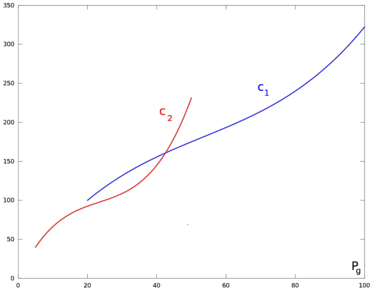
and, for completeness,

$$k_{g1} = 6.775, \quad k_{g2} = -0.101, \quad k_{g3} = 5.917 \cdot 10^{-4}.$$

- Note: r_{fP} can be interpreted as the *inverse* of the fuel-to-power *efficiency* $\eta_{fP}(P_g) = P_g/Q_f(P_g)$, ranging in this case from 0.4 (optimal point) to 0.2 (minimum sustainable load).

- Generator data:

$$
\begin{array}{lllll}
P_{g1,max} = 100 & P_{g1,min} = 20 & P^o_{g1} = 80 & r^o_{fP1} = 3 & r^{ml}_{fP1} = 5 \\
P_{g2,max} = 50 & P_{g2,min} = 5 & P^o_{g2} = 35 & r^o_{fP2} = 3.5 & r^{ml}_{fP2} = 8
\end{array}
$$



- Network power demand: $P_{e,max} = 130$, $P_{e,min} = 10$.
- We suppose that generators can be activated or deactivated (almost) instantaneously and at no cost (a very strong hypothesis on which we shall return later on).

- Basic idea (which is general w.r.t. the example):
  - take the forecast power demand $\widehat{P}_e$ for the next "period" (day, hour,...),
  - determine the optimal generation distribution $\{P_{gi}^{opt}\}$ yielding $\widehat{P}_e$ at minimum cost,
  - send each of the so obtained generation requests P_{gi}^{opt} to the corresponding generator as a bias value,
  - and let primary and secondary control act as usual.
- Let us now concentrate on the $\widehat{P}_e \mapsto \{P_{gi}^{opt}\}$ problem, other aspects later on.

An introductory case
Two generators

- Consider all the generator combinations, and determine the minimum and maximum power that can be generated by each of them:

$$G1 \to [20, 100], \quad G2 \to [5, 50], \quad G1 + G2 \to [25, 150].$$

- Consequently, divide the $\widehat{P}_e$ range in intervals I_i and determine the feasible combinations for each of said intervals (easier to show than to explain):

```
                 20                             100
G1               |-------------------------------|
           5                50
G2         |-----------------|
                    25                                                150
G1+G2               |-------------------------------------------------|
```

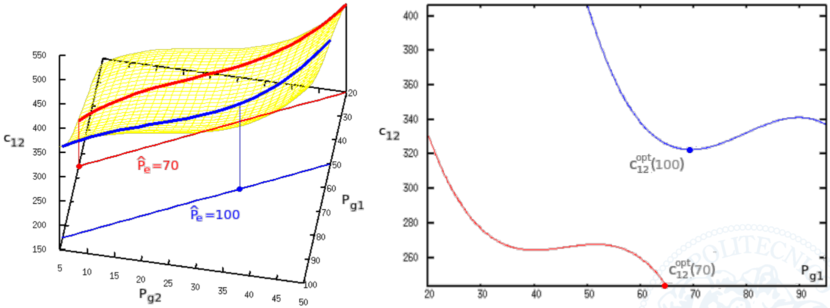
I_1	$[10, 20]$	$\to$	G2
I_2	$[20, 25]$	$\to$	G1,G2
I_3	$[25, 50]$	$\to$	G1,G2,G1+G2
I_4	$[50, 100]$	$\to$	G1,G1+G2
I_5	$[100, 130]$	$\to$	G1+G2

- For combinations with more than one generator, the optimal distribution (minimum total cost) has to be found.
- With only two generators the only case to consider is G1+G2, and we can proceed by substitution (we shall see something more general later on):

$$P_{g2} = \widehat{P}_e - P_{g1} \Rightarrow c_{12}(P_{g1}) = c_1(P_{g1}) + c_2(\widehat{P}_e - P_{g1})$$

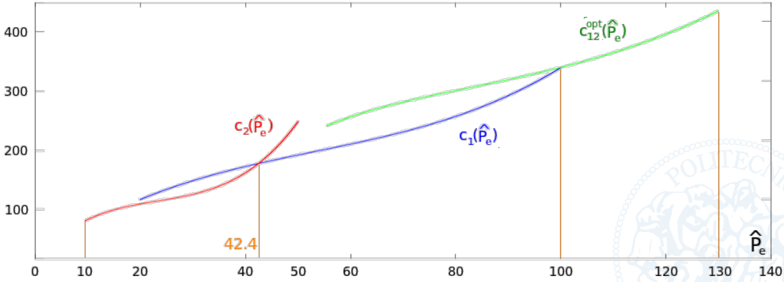
- Then we take the first and second derivative of $c_{12}(P_{g1}$ w.r.t. the only remaining independent variable P_{g1},
- find a possible minimum cost $c_{12}^{opt}(\widehat{P}_e)$ inside the power range of both generators (otherwise the minimum is at one of the two distribution extrema),
- determine the power distribution – i.e., $P_{g1}^{opt}(\widehat{P}_e)$ – corresponding to the minimum.

- Graphical interpretation for the G1+G2 combination:



- As can be seen, for a given $\widehat{P}_e$, the optimal distribution can be inside the segment of the $P_{g1} + P_{g2} = \widehat{P}_e$ straight line, or at one of its extrema.

- As illustrated by the previous graphical interpretation, for each value of $\widehat{P}_e$, the combination can now be chosen that provides the minimum cost; this is shown below:



- Finally, based on the choice just made, the bias (or "tertiary control") values $P_{b1,2}$ for $P_{g1,2}$ are determined. We omit this part as we shall see it in the following examples and exercises.
- Let us now quit this introductory example and the naïve technique used to address it, moving toward establishing a methodology.
- To this end, we split the problem in three:
  - determining the optimal generation distribution given the active generators,
  - integrating tertiary control with primary/secondary control, and discussing the mutual influences.
  - determine *both* the active generators and the distribution (just a few words on this),
- Before entering the subject, however, we need to briefly review some mathematics.

Problem statement

- We want to minimise a real function f of N_x real variables x_i, i.e.,

$$f(x_1, x_2, \dots x_{N_x}), \qquad f(\cdot, \cdot, \dots \cdot) \in \Re, \;\; x_i \in \Re, \; i = 1 \dots N_x,$$

  subject to N_e equality constraints in the form

$$g_i(x_1, x_2, \dots x_{N_x}) = 0, \qquad g_i(\cdot, \cdot, \dots \cdot) \in \Re, \; i = 1 \dots N_e,$$

  and to N_i inequality constraints in the form

$$h_i(x_1, x_2, \dots x_{N_x}) \geq 0, \qquad h_i(\cdot, \cdot, \dots \cdot) \in \Re, \; i = 1 \dots N_i.$$

Caveat: this is *not* a math lecture. We shall implicitly assume that "everything is regular enough", and not even mention several hypotheses that would be necessary for a rigorous treatise. We just want to understand the methods' operation and significance in our context.

- Form the problem's *Lagrangian* as

$$L = f(x_1, x_2, \dots x_{N_x}) + \sum_{i=1}^{N_e} \lambda_i g_i(x_1, x_2, \dots x_{N_x})$$

  introducing N_e additional real unknowns λ_i, named the *Lagrange multipliers*.
- Compute the gradients of L w.r.t. vectors $x = [x_1 \dots x_{N_x}]' \in \Re^{N_x}$ and $\lambda = [\lambda_1 \dots \lambda_{N_e}]' \in \Re^{N_e}$, i.e.,

$$\nabla_x L(x,\lambda) = \left[\frac{\partial L}{\partial x_1}\frac{\partial L}{\partial x_2}\dots\frac{\partial L}{\partial x_{N_x}}\right], \qquad \nabla_\lambda L(x,\lambda) = \left[\frac{\partial L}{\partial \lambda_1}\frac{\partial L}{\partial \lambda_2}\dots\frac{\partial L}{\partial \lambda_{N_e}}\right],$$

  having respectively N_x and N_e (function) components.

- Observe that the k-th component of $\nabla_x L$ is

$$\frac{\partial L}{\partial x_k} = \frac{\partial f}{\partial x_k} + \sum_{i=1}^{N_e} \lambda_i \frac{\partial g_i}{\partial x_k} = \frac{\partial f}{\partial x_k} + \lambda' \cdot \begin{bmatrix} \frac{\partial g_1}{\partial x_k} \\ \vdots \\ \frac{\partial g_{N_e}}{\partial x_k} \end{bmatrix}$$

  where $\cdot$ denotes the scalar product. Therefore

$$\nabla_x L = \nabla_x f + \lambda' \cdot \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \cdots & \frac{\partial g_1}{\partial x_{N_x}} \\ \vdots & & \vdots \\ \frac{\partial g_{N_e}}{\partial x_1} & \cdots & \frac{\partial g_{N_e}}{\partial x_{N_x}} \end{bmatrix} = \nabla_x f + \lambda' \cdot \begin{bmatrix} \nabla_x g_1 \\ \vdots \\ \nabla_x g_{N_e} \end{bmatrix} = \nabla_x f + \lambda' J_x g$$

  where $J_x g$ is the Jacobian of the constraints g w.r.t. x.
- Also, observe that the k-th component of $\nabla_\lambda L$ is g_k.

Brief math review

Only equality constraints – Lagrange multipliers

- Now, suppose that (x^o, λ^o) is a solution for the system of $N_x + N_e$ equations

$$\begin{cases} \nabla_x L(x,\lambda) & = & 0_{1\times N_x} \\ \nabla_\lambda L(x,\lambda) & = & 0_{1\times N_e} \end{cases}$$

  in the $N_x + N_e$ unknowns (x, λ), termed the Lagrangian Multipliers (LM) equations.
- The second equation above tells us that vector x^o fulfils the constraints $g(x)$. Let us now consider the other equation, and distinguish two cases.
- Case 1: $\nabla_x f$ in x^o is a zero vector.
  - In this case x^o is a *stationary point* for function $f(x)$ *independently of the presence of the constraints* $g(x)$.
  - In addition, given the expression of $\nabla_x L$, the equation reduces to $\lambda' J_x g = 0$, and since (x^o, λ^o) fulfils it, either λ^o is a zero vector, or the gradients $\nabla_x g_i$, $i = 1 \ldots N_e$, are linearly dependent when evaluated in x^o.
  - If the $\nabla_x g_i$ are linearly dependent in x^o we shall then say that x^o *may* not be a *regular point* for the constraints g; however, for the problems encountered here, we are not interested in analysing this situation: details are in specialised courses.

- Case 2: $\nabla_x f$ in x^o is not a zero vector.
  - In this case λ^o cannot be a zero vector either, or the considered equation $\nabla_x f + \lambda' J_x g = 0$ cannot be satisfied (contrary to the hypothesis).
  - Also, rewritten as $\nabla_x f = -\lambda' J_x g$, the same equation tells us that the gradients of $f(x)$ and $g(x)$ w.r.t. x are parallel in x^o.
  - Let now $z^o = f(x^o)$, and consider the hypercurve in $\Re^{N_x+1}$ obtained by intersecting the hypersurfaces $z = f(x)$ and $g(x) = 0$. Moving on that hypercurve away from (x^o, z^o), that apparently belongs to it, *locally* produces no variation of z. Therefore, x^o is a (local) stationary point for $f(x)$ constrained by $g(x) = 0$.
  - Since this may be hard to grasp, let us see an example with Maxima:

```
f : x1^2+x2^2;
g : x1-1;                                  /* grad_x g = [1 0]         */
L : f+lam*g;
solve([diff(L,x1),diff(L,x2),
       diff(L,lam)],[x1,x2,lam]);
plot3d([f,g,0, [x1,0,2], [x2,-1,1]]);
subst([x1=1,x2=0],jacobian([f],[x1,x2])); /* grad_x f || grad_x g in xo */
subst([x1=1,x2=1],jacobian([f],[x1,x2])); /* and not e.g. here          */
```

- Conclusion: x^o fulfils the LM equations
  $\Rightarrow$ it is a *candidate* constrained optimal point.
- We have then found a set of *necessary*, *first-order* conditions for *local* constrained optimality (enough for us in this course).
- Homework: *positively* review your notes, next time ask questions if needed.

Generation cost optimisation

A methodology and some considerations (cont'd)

- Recap
  - We want to minimise a real function f of N_x real variables x_i, i.e.,

    $$f(x_1, x_2, \ldots x_{N_x}), \qquad f(\cdot, \cdot, \ldots \cdot) \in \Re, \;\; x_i \in \Re, \; i = 1 \ldots N_x,$$

    subject to N_e equality constraints in the form

    $$g_i(x_1, x_2, \ldots x_{N_x}) = 0, \qquad g_i(\cdot, \cdot, \ldots \cdot) \in \Re, \; i = 1 \ldots N_e,$$

    (ie.e, no inequality constraints for the moment).
  - We form the Lagrangian

    $$L(x, \lambda) = f(x_1, x_2, \ldots x_{N_x}) + \sum_{i=1}^{N_e} \lambda_i g_i(x_1, x_2, \ldots x_{N_x}) = f(x) + \lambda \cdot g(x)$$

    where x and λ are vectors, and g the vector of functions g_i.
  - We solve

    $$\begin{cases} \nabla_x L(x, \lambda) & = & 0_{1 \times N_x} \\ \nabla_\lambda L(x, \lambda) & = & 0_{1 \times N_e} \end{cases}$$

    for x and λ, which provides the stationary points x for f constrained by g (overlooking quite a bit of mathematical details not relevant for our use of the method) and the Lagrange multipliers at said points.

Brief math review

Only equality constraints – Lagrange multipliers

- Next question: is x^o a minimum, a maximum, or neither?
- The matter may get *very* tricky; here we limit our scope to analysing the *Hessian* (matrix) of L, which we know to be an extension of the second-derivative (concavity) test in the univariate case.
- The Hessian is the second derivatives' matrix; let us see what its elements look like in our problem:

$$
\begin{aligned}
L_{x_i x_j}(x,\lambda) &:= \frac{\partial^2 L(x,\lambda)}{\partial x_i x_j} &&= \frac{\partial^2 f(x)}{\partial x_i x_j} + \lambda \cdot \frac{\partial^2 g(x)}{\partial x_i x_j} && && = f_{x_i x_j} + \lambda \cdot g_{x_i x_j}, \\
L_{x_i \lambda_j}(x,\lambda) &:= \frac{\partial^2 L(x,\lambda)}{\partial x_i \lambda_j} &&= \frac{\partial}{\partial \lambda_j}\left(\frac{\partial f(x)}{\partial x_i} + \lambda \cdot \frac{\partial g(x)}{\partial x_i}\right) && = \frac{\partial g_j(x)}{\partial x_i} && = g_{j x_i}, \\
L_{\lambda_i \lambda_j}(x,\lambda) &:= \frac{\partial^2 L(x,\lambda)}{\partial \lambda_i \lambda_j} &&= 0
\end{aligned}
$$

- Note that we are assuming "everything regular enough", so e.g. mixed derivatives are continuous, hence the order of differentiation in them is irrelevant and the Hessian is symmetric.

- We can now form the Hessian, starting from the $\lambda\lambda$ derivatives:

$$H_{x,\lambda}L = \begin{bmatrix} L_{\lambda_1\lambda_1} & \cdots & L_{\lambda_1\lambda_{N_e}} & L_{\lambda_1 x_1} & \cdots & L_{\lambda_1 x_{N_x}} \\ \vdots & & \vdots & \vdots & \vdots & \vdots \\ L_{\lambda_{N_e}\lambda_1} & \cdots & L_{\lambda_{N_e}\lambda_{N_e}} & L_{\lambda_{N_e} x_1} & \cdots & L_{\lambda_{N_e} x_{N_x}} \\ L_{x_1\lambda_1} & \cdots & L_{x_1\lambda_{N_e}} & L_{x_1 x_1} & \cdots & L_{x_1 x_{N_x}} \\ \vdots & & \vdots & \vdots & \vdots & \vdots \\ L_{x_{N_x}\lambda_1} & \cdots & L_{x_{N_x}\lambda_{N_e}} & L_{x_{N_x} x_1} & \cdots & L_{x_1 x_{N_x}} \end{bmatrix}$$

- and then, bringing previous expressions and symmetry in, we have

$$H_{x,\lambda}L = \begin{bmatrix} 0 & \cdots & 0 & g_{1_{x_1}} & \cdots & g_{1_{x_{N_x}}} \\ \vdots & & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & g_{N_{e\,x_1}} & \cdots & g_{N_{e\,x_{N_x}}} \\ g_{1_{x_1}} & \cdots & g_{N_{e\,x_1}} & f_{x_1 x_1} + \lambda \cdot g_{x_1 x_1} & \cdots & f_{x_1 x_{N_x}} + \lambda \cdot g_{x_1 x_{N_x}} \\ \vdots & & & \vdots & & \vdots \\ g_{1_{x_{N_x}}} & \cdots & g_{N_{e\,x_{N_x}}} & f_{x_{N_x} x_1} + \lambda \cdot g_{x_{N_x} x_1} & \cdots & f_{x_{N_x} x_{N_x}} + \lambda \cdot g_{x_{N_x} x_{N_x}} \end{bmatrix}$$

- As can be seen, the first N_e principal minors of $H_{x,\lambda}L$, that for the sake of precision is called a *bordered* Hessian, are zero.
- Thus to apply the Hessian test we only need to compute the determinant of N_x principal minors.
- The test is then as follows (we do not have the space for any proof):
  - let x^o be a stationary point for f constrained by g, and λ^o the corresponding Lagrange multiplier vector;
  - let $\{m_i\}$ be the set of the N_x nonzero principal minor determinants of $H_{x,\lambda}L$ evaluated in (x^o, λ^o);
  - if the m_i alternate in sign, and the sign of the last (the Hessian itself's determinant) is that of $(-1)^{N_x}$, then x^o is a (local constrained) maximum;
  - if the m_i have all the same sign, and that sign is that of $(-1)^{N_e}$, then x^o is a (local constrained) minimum;
  - otherwise the stationary point is neither a maximum nor a minimum (and we are not interested in further investigations).

- Find the minimum and the maximum height (z coordinate) of the surface $z(x, y) = x^2 + 2y^2$ constrained by $(x - 2)^2 + (y - 3)^2 = 1$.
- Hint 1: *first* formulate the problem on paper, *then* use Maxima.
- Hint 2: a geometric representation of the problem may help understand what we are doing.

- Solve the problem (5 minutes), then we work it out together.

Brief math review (cont'd)

Only equality constraints – Lagrange multipliers – a final remark

- The Lagrange multipliers λ_i have an interesting interpretation as "shadow cost" for the constraints.
- More precisely, the value of the Lagrange multiplier for a specific constraint g_i is the rate at which the optimal value of the objective function f changes if that constraint is changed from $g_i(x) = 0$ to $g_i(x) = \alpha_i$, α_i being "very" (rigorously, *infinitely*) close to zero.
- This has a significant relevance for example when one can decide to input power in a network by *generating* or *purchasing* that power: if a company can purchase power for a price less than the shadow cost of the power balance constraint, then doing so will reduce their overall cost.

- Problem statement (recap)
  - We want to minimise a real function f of N_x real variables x_i, i.e.,

$$f(x_1, x_2, \dots x_{N_x}), \qquad f(\cdot, \cdot, \dots \cdot) \in \Re, \;\; x_i \in \Re, \; i = 1 \dots N_x,$$

subject to N_e equality constraints in the form

$$g_i(x_1, x_2, \dots x_{N_x}) = 0, \qquad g_i(\cdot, \cdot, \dots \cdot) \in \Re, \; i = 1 \dots N_e,$$

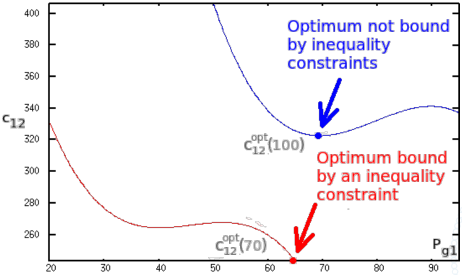
and to N_i inequality constraints in the form

$$h_i(x_1, x_2, \dots x_{N_x}) \geq 0, \qquad h_i(\cdot, \cdot, \dots \cdot) \in \Re, \; i = 1 \dots N_i.$$

Brief math review (cont'd)

Equality and inequality constraints – Karush-Kuhn-Tucker (KKT) equations

- Major difference w.r.t. the equality-only (LM) case: a solution may *not* be a stationary point, as some inequality constraints may bind it.
- Note that we have already found such a case in the introductory example with two generators:



- Hence the LM equations cannot be used here (i.e., the Lagrange *rationale* still works, but we need to introduce some modifications).

- We form again the Lagrangian, this time however in the form

$$\begin{aligned} L(x,\lambda,\mu) &= f(x) + \sum_{i=1}^{N_e} \lambda_i g_i(x) + \sum_{j=1}^{N_i} \mu_j h_j(x) \\ &= f(x) + \lambda \cdot g(x) + \mu \cdot h(x) \end{aligned}$$

  where another multiplier vector $\mu = [\mu_1 \ldots \mu_{N_i}]' \in \Re^{N_i}$ is introduced w.r.t. the case with equalities only, and $h \in \Re^{N_i}$ is the vector of functions h_j,
- and give the following simple definitions:
  - if at a certain point x^o a certain inequality constraint h_j is satisfied with equality – i.e., if $h_j(x^o) = 0$ – we shall say that the constraint h_j is *active* (or *binding*) in x^o;
  - if the constraint is satisfied with the $>$ sign – i.e., if $h_j(x^o) > 0$ – we shall say that it is *inactive* (or *nonbinding*) in x^o;
  - otherwise (obviously) the constraint is *violated* in x^o.

- Now (we again look for necessary conditions) suppose that x^o is a solution (i.e., an optimal point) with neither binding nor violated inequality constraint, i.e., that $h_j(x^o) > 0\,\forall j$.
- In this case x^o is also a solution for the LM problem, as setting $\mu = 0$ makes the term $\mu \cdot h(x^o)$ contribute zero to L.
- Note that also a solution for the LM problem *violating* some inequality constraint would fall in the same case, but it is not difficult to see if said constraints are violated or nonbinding. In the following we assume that such a *feasibility check* is always performed.
- The most interesting case is when at least one inequality constraint is binding. Let us expand a bit on this.

Brief math review (cont'd)

Equality and inequality constraints – Karush-Kuhn-Tucker (KKT) equations

- By adopting the same notation introduced in the LM problem, consider the system $N_x + N_e + N_i$ equations

$$\begin{cases} \nabla_x L(x,\lambda,\mu) & = & \nabla_x f(x) + \lambda' J_x g(x) + \mu' J_x h(x) & = & 0 \\ \nabla_\lambda L(x,\lambda,\mu) & = & g(x) & = & 0 \\ \mu' \circ \nabla_\mu L(x,\lambda,\mu) & = & \mu' \circ h(x) & = & 0 \end{cases}$$

  in the $N_x + N_e + N_i$ unknowns (x, λ, μ), where $\circ$ denotes the Schur (element by element) product.

- Roughly speaking, a solution bound by some inequality constraints will satisfy a LM problem where said constraints are fictitiously treated as equality ones (whence the last term in the first equation) provided that only those (binding) constraints are actually accounted for, which is ensured by the third equation (of course s.t. the necessary feasibility checks).

- The system above is composed of the so-called KKT equations, and is of course subject to the feasibility condition $h(x) \geq 0$. We do not delve into the involved mathematics anymore.

Brief math review (cont'd)

Equality and inequality constraints – Karush-Kuhn-Tucker (KKT) equations

- Just a further remark: if a constraint $h_i(x) \geq 0$ is inactive *at optimality*, i.e. in x^o, then the corresponding μ_i zero.
- In the opposite case, the sign of μ_i dictates whether f increases or decreases when entering or exiting the admissible region as dictated by the inequality constraints.
- Assuming that we want to *minimise* $f(x)$ and the sign in the $h(x)$ equality constraints is $\geq$, requiring that $f(x)$ *increase* when x *enters* the feasibility region for all binding h_i – i.e., that a KKT solution with at least one binding inequality constraint be a candidate *bound minimum* – corresponds to requiring that all nonzero μ_i be *negative* in it. Of course all the other combinations are possible (we may want to maximise $f(x)$ and inequality constraints may have the $\leq$ sign).
- Let us now go for two elementary examples.

- Minimise $f(x) = x^2$ s.t. $x \geq 1$ ($\Rightarrow x - 1 \geq 0$), $x \leq 2$ ($\Rightarrow 2 - x \geq 0$).
- Maxima;

```
f      : x^2;
h1     : x-1;
h2     : 2-x;
L      : f+mu1*h1+mu2*h2;
KKTeqs : [diff(L,x),mu1*diff(L,mu1),mu2*diff(L,mu2)];
solve(KKTeqs,[x,mu1,mu2]);
```

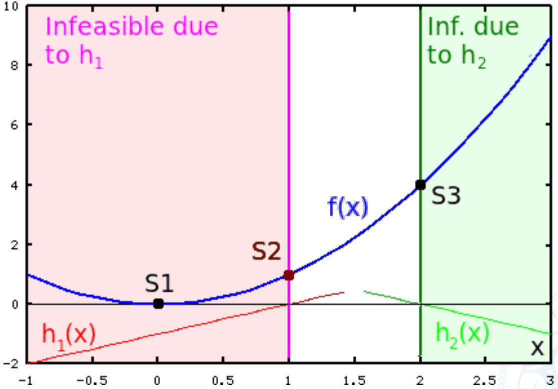
- Solutions found:

	x	μ_1	μ_2	$f(x)$	$h_1(x)$	$h_2(x)$
S1	0	0	0	0	-1	2
S2	1	-2	0	1	0	1
S3	2	0	4	4	1	0

- S1: infeasible.
- S2: feasible, h_2 nonbinding, h_1 binding and entering the feasible region increases f $\Rightarrow$ can be a bound minimum.
- S3: feasible, h_1 nonbinding, h_2 binding and entering the feasible region decreases f $\Rightarrow$ cannot be a bound minimum.

- Hence, the solution is S2.

- This was *really* elementary, so let us have a visual look:



- S1: infeasible.
- S2: feasible, h_2 nonbinding, h_1 binding and entering the feasible region increases f $\Rightarrow$ can be a bound minimum.
- S3: feasible, h_1 nonbinding, h_2 binding and entering the feasible region decreases f $\Rightarrow$ cannot be a bound minimum.

Equality and inequality constraints – KKT equations – elementary example 2

- Minimise $f(x, y) = x^2 + y^2$ s.t. $x + y = 1$, $x \geq 0.1$, $y \geq 0.2$.
- Maxima;

```
f      : x^2+y^2;
g      : 1-x-y;
h1     : x-0.1;
h2     : y-0.2;
L      : f+lambda*g+mu1*h1+mu2*h2;
KKTeqs : [diff(L,x),diff(L,y),
          diff(L,lambda),
          mu1*diff(L,mu1),mu2*diff(L,mu2)];
solve(KKTeqs,[x,y,lambda,mu1,mu2]);
```

Brief math review (cont'd)

Equality and inequality constraints – KKT equations – elementary example 2

- Solutions found:

	x	y	λ	μ_1	μ_2	$f(x,y)$	$g(x,y)$	$h_1(x,y)$	$h_2(x,y)$
S1	0.5	0.5	1.0	0.0	0.0	0.50	0	0.4	1.5
S2	0.1	0.9	1.8	1.6	0.0	0.82	0	0.0	1.1
S3	0.8	0.2	1.6	0.0	1.2	0.68	0	0.7	1.8

- All feasible, no candidate bound minimum.
- The only candidate is S1, with no binding inequality. We thus need to check the bordered Hessian:

```
H: matrix([0         , diff(g,x)      , diff(g,y)],
          [diff(g,x), diff(L,x,2)    , diff(L,x,1,y,1)],
          [diff(g,y), diff(L,y,1,x,1), diff(L,x,2)]);
```

- We obtain

$$H = \begin{bmatrix} 0 & -1 & -1 \\ -1 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$

Note: in general H depends on (x, λ), this is a very particular case.

- Two principal minors to check (recall that in general the values x^o and λ^o need substituting into H):

$$m_1 = \det \begin{bmatrix} 0 & -1 \\ -1 & 2 \end{bmatrix} = -1, \qquad m_2 = \det \begin{bmatrix} 0 & -1 & -1 \\ -1 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix} = -4.$$

- All negative, and since there is one equality constraint ($N_e = 1$) the sign of $(-1)^{N_e}$ is negative too.
- Thus S1 provides the sought minimum.

Back to generation optimisation

The previous two-generator case revisited with the KKT equations

- We are of course looking at the G1+G2 combination, thus the problem is
- minimise $f(P_{g1}, P_{g2}) = c_1(P_{g1}) + c_2(P_{g2})$
  s.t. $P_{g1} + P_{g2} = \widehat{P}_e$, $P_{g1,min} \leq P_{g1} \leq P_{g1,max}$, $P_{g2,min} \leq P_{g2} \leq P_{g2,max}$.
- Maxima (preliminaries):

```
Pg1min : 20;
Pg1max : 100;
Pgo1   : 80;
ro1    : 3;
rml1   : 5;

Pg2min : 5;
Pg2max : 50;
Pgo2   : 35;
ro2    : 3.5;
rml2   : 8;

k11    : (rml1*Pgo1^2-ro1*Pg1min*(2*Pgo1-Pg1min))/(Pgo1-Pg1min)^2;
k21    : 2*Pgo1*(ro1-rml1)/(Pgo1-Pg1min)^2;
k31    : (rml1-ro1)/(Pgo1-Pg1min)^2;
k12    : (rml2*Pgo2^2-ro2*Pg2min*(2*Pgo2-Pg2min))/(Pgo2-Pg2min)^2;
k22    : 2*Pgo2*(ro2-rml2)/(Pgo2-Pg2min)^2;
k32    : (rml2-ro2)/(Pgo2-Pg2min)^2;

c1     : k11*Pg1+k21*Pg1^2+k31*Pg1^3;
c2     : k12*Pg2+k22*Pg2^2+k32*Pg2^3;
f      : c1+c2;
```

Back to generation optimisation

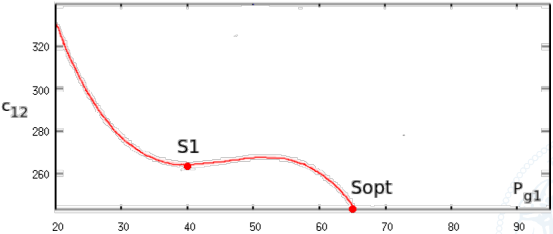
The previous two-generator case revisited with the KKT equations

- Maxima (we consider the two cases with $\widehat{P}_e$ equal to 70 and 100):

```
/* Set Pe to the desired value, 70 or 100 for the two cases shown */
Pe      : 70;

g       : Pg1+Pg2-Pe;
h1      : Pg1-Pg1min;
h2      : Pg1max-Pg1;
h3      : Pg2-Pg2min;
h4      : Pg2max-Pg2;

L       : f+lambda*g+mu1*h1+mu2*h2+mu3*h3+mu4*h4;

KKTeqs : [diff(L,Pg1),diff(L,Pg2),
          diff(L,lambda),
          mu1*diff(L,mu1),mu2*diff(L,mu2),mu3*diff(L,mu3),mu4*diff(L,mu4)];

S       :  solve(KKTeqs,[Pg1,Pg2,lambda,mu1,mu2,mu3,mu4]);

for i:1 thru length(%rnum_list) do S:subst(t[i],%rnum_list[i],S);
float(S);
fvals   : float(makelist(subst(S[i],f),i,1,length(S)));
```

- Results for $\widehat{P}_e = 70$:

P_{g1}	P_{g2}	λ	μ_1	μ_2	μ_3	μ_4	f	
20.00	50.00	any	$-\lambda - \frac{11}{3}$	0.00	0.00	$\lambda + \frac{97}{8}$	331.25	left extremum
51.58	18.42	-1.82	0.00	0.00	0.00	0.00	267.67	local max
40.09	29.91	-2.11	0.00	0.00	0.00	0.00	264.30	S1
100.00	-30.00	-44.12	0.00	-38.68	0.00	0.00	-416.52	infeasible
65.00	5.00	-2.04	0.00	0.00	-4.46	0.00	243.13	Sopt



- Note that the left extremum cannot be a bound minimum, while the right one actually is.
- Homework: check and comment (as was done here) the case $\widehat{P}_e = 100$.

Lecture 11 (2P)

Previous homework solution

Classroom practice

Power and frequency control
- primary, secondary and tertiary regulation -

Some words on the active generators' pool determination

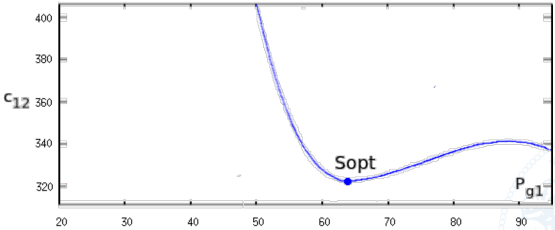
Previous homework solution

Previous homework solution

The two-generator case revisited with the KKT equations

- Results for $\widehat{P}_e = 100$:

P_{g1}	P_{g2}	λ	μ_1	μ_2	μ_3	μ_4	f	
89.75	10.25	-4.02	0.00	0.00	0.00	0.00	341.26	local max
69.42	30.58	-2.25	0.00	0.00	0.00	0.00	322.59	Sopt
20.00	80.00	-49.63	45.95	0.00	0.00	0.00	1190.00	infeasible
100.00	0.00	-9.63	0.00	-4.18	0.00	0.00	322.22	infeasible
95.00	5.00	-4.71	0.00	0.00	-1.79	0.00	336.88	right extremum
50.00	50.00	-1.83	0.00	0.00	0.00	10.29	406.25	left extremum



- Note that the left extremum cannot be a bound minimum, while the right one could but is not.
- Never forget that the KKT equations not only are necessary conditions, but just provide *candidate optima*: always check the solutions!

Classroom practice

Power and frequency control

- primary, secondary and tertiary regulation -

Lessons to learn

i.e., just one slide for this lecture's objectives

- Understand the interactions of primary/secondary control and power scheduling (tertiary) control.
- Briefly discuss the possible objectives to pursue, so as to appreciate the problem complexity.
- Envisage the possible (and somehow expected) impact of massive distributed generation.

- Note that this covers the second of the three parts in which we have split the problem.

Some words on the active generators' pool determination

Just a sketch of the problem

i.e., the third of the three parts in which we have split the original one

- The *scenario* can be very complex, also involving contractual/regulatory facts that of course do not fit in this treatise.
- In extreme synthesis, two *extrema* can be envisaged;
  - the pool is pre-determined for each period, for example as per stipulated contracts, and thus is taken by any optimisation as an *a priori* information;
  - for each generator thet is inactive (active) when the optimisation problem is to be solved, the time needed for activating (deactivating) it is known or estimated, together with the cost of the activation (deactivation) operation.
- The second situation makes the problem very challenging, but we cannot even scratch its surface. Knowing the *panorama* illustrated so far is considered enough for this course.

Lecture 12 (2L)

Load flow

- The basics
- Problem formulation and solution method

The basics

Problem statement

Preliminaries

- Prior to entering the subject, we need to review two basic concepts:
  - how power is transferred from a generator to the network $\Rightarrow$ machine angle,
  - and how the effects of generators are combined $\Rightarrow$ network admittance matrix.
- Since we are dealing with AC networks, we need to abandon the purely energetic approach (no voltages or currents) taken so far, and adopt a phasor-oriented vision.
- Without impairing the conveyed message, we also adopt *for the target of this course* the following simplifications:
  - we have a single-voltage network (no transformers),
  - we consider a single-ohase (or equivalently, a perfectly balanced mutiphase) system,
  - the amplitude of the voltage produced by each generator is controlled ideally,
  - and finally we assume a *prevailing network*, i.e., one in which all the generators are individually so small compared to the union of the others that each of them sees the network voltage as a fixed phasor.
- Of course the network frequency is controlled (we now know how).

- Let $\underline{V}_n = V$ (phase 0) be the network voltage phasor (recall that complex numbers are underlined).
- Suppose that the generator voltage has amplitude controlled to be V, thus taking the form
$$\underline{V}_g = V\,(\cos\delta + j\sin\delta)$$
where δ is the *machine angle* (w.r.t. the network).
- Let finally $\underline{Y}_{gn} = G_{gn} - jB_{gn}$ – mind the minus! – be the admittance of the generator–network connection.
- Then, some Maxima gives us the active and reactive power (P and Q, respectively) flowing from the generator to the network:

```
Vn  : V;
Vg  : V*cos(d)+%i*V*sin(d);
Ygn : G-%i*B;
Ign : (Vg-Vn)*Ygn;
Sgn : Vg*conjugate(Ign);
P   : trigsimp(realpart(Sgn));
Q   : trigsimp(imagpart(Sgn));
```

- We have
$$\begin{aligned} P &= (G(1-\cos\delta)+B\sin\delta)\,V^2, \\ Q &= (B(1-\cos\delta)-G\sin\delta)\,V^2. \end{aligned}$$
- Thus, varying δ one can control P, and Q will follow as a consequence.
- To control *both* P and Q one may for example act on the excitation voltage. To see that, we can redo the same computations with the amplitude of $\underline{V}_g$ set to V_g instead of V, and we obtain
$$\begin{aligned} P &= GV_g^2+(B\sin\delta-G\cos\delta)\,V\,V_g, \\ Q &= BV_g^2-(G\sin\delta+B\cos\delta)\,V\,V_g. \end{aligned}$$
- Reactive power in a network is controlled in many other ways. In this course we do not deal with reactive power control.
- Suffice thus here to say that to govern its power transfer to the network, a generator varies its δ with *transient* accelerations or decelerations...
- ...that however do not influence the network frequency, thus $\underline{V}_n$, given the prevailing network hypothesis.

- So far, we have been dealing with the problem of optimising the generation of the required power.
- However, generating some power here or there requires to use the transmission lines in a different manner.
- We need to ensure that none of them gets overloaded, and if possible to account – when computing the generation cost – also for the power lost in the transmission process itself.
- In other words, we need to check how power *flows* in the network, and this problem has been historically given two names:
  - the *Load Flow* (LF) problem, on which we are going to say some words,
  - and the *Optimal Power Flow* (OPF) problem, that we shall just mention.
- Before entering LF, however, we need to revise the concept of *(network) admittance matrix*.

The basics

Network-admittance matrix (and some definitions)

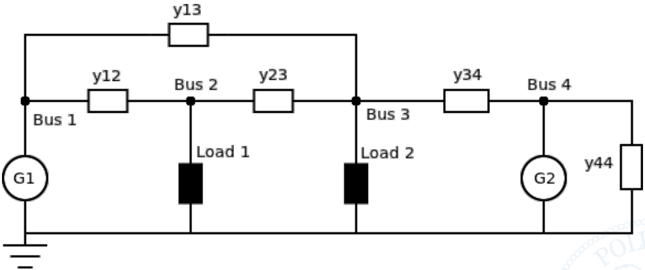
- Consider a network with n_B nodes or, to adopt the specific jargon of the addressed problems, n_B *busses*. Let $\underline{V}_i$ be the voltage at bus i and $\underline{I}_i$ the current injected in it by the locally connected generator(s) or drawn from it by the (local) bus load(s). Always recall that we are operating with *phasors*.
- In general each bus is connected to others via lines. Let $\underline{y}_{ij} = g_{ij} - jb_{ij}$ be the complex admittance of the line connecting busses i and j, of course with $i \neq j$ (the reason for using lowercase letters here will become clear very soon).
- Also, a bus may exhibit an admittance to ground. If this is true for bus i, we shall denote that admittance by $\underline{y}_{ii} = g_{ii} - jb_{ii}$.
- Finally, some busses have at least one generator attached to them, and will be termed *Generator* (G) busses. The other busses carry only loads that absorb a certain amount of active power P and reactive power Q; these are called *Load* (L) busses or, more frequently, PQ busses.
- We need to describe the network by a matrix that will then be useful for LF, and is called the admittance matrix.

- The admittance matrix $\underline{\mathbf{Y}}$ is created by
  - starting from the usual representation of a network containing *voltage* generators and *impedances* (there is a more synthetic formalism used in power network engineering, called the *one-* or *single-line diagram*, but we do not have the time to treat it),
  - injecting a *current* $\underline{I}_i$ in each bus i and computing the so induced voltages $\underline{V}_j$, in all the nodes,
  - indicating by $\underline{Y}_{ij}$ (uppercase, notice) the $\underline{I}_i/\underline{V}_j$ ratio,
  - and finally assembling $\mathbf{Y}$ as

$$\underline{\mathbf{Y}} = [\underline{Y}_{ij}] = \begin{cases} \underline{y}_{ii} + \sum_{j=1, j\neq i}^{n_B} \underline{y}_{ij} & \text{for the diagonal elements,} \\ -\underline{y}_{ij} & \text{for the other elements,} \end{cases}$$

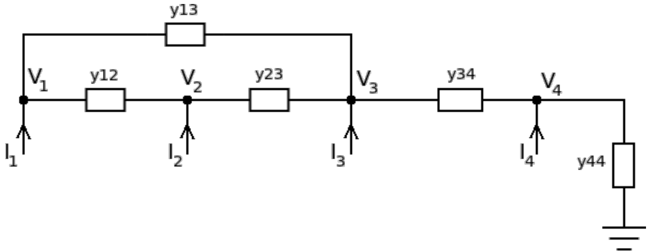
  - It should now be clear why we have used lowercase letters. Let us see an example to confirm our comprehension.

- Consider the 4-busses network shown below:



- Remove all generators and loads, denote by $\underline{V}_i$ the voltage at the i-th bus, inject in each bus a current $\underline{I}_i$, write the nodal equations (KCL), and solve for the injected nodal currents. You have three minutes, then we do it together.

- Network with bus voltages and injected currents:



- Nodal equations (KCL) for busses 1–4:

$$
\begin{aligned}
\underline{I}_1 &= \underline{y}_{12}\left(\underline{V}_1 - \underline{V}_2\right) + \underline{y}_{13}\left(\underline{V}_1 - \underline{V}_3\right) \\
\underline{I}_2 &= \underline{y}_{12}\left(\underline{V}_2 - \underline{V}_1\right) + \underline{y}_{23}\left(\underline{V}_2 - \underline{V}_3\right) \\
\underline{I}_3 &= \underline{y}_{23}\left(\underline{V}_3 - \underline{V}_2\right) + \underline{y}_{13}\left(\underline{V}_3 - \underline{V}_1\right) + \underline{y}_{34}\left(\underline{V}_3 - \underline{V}_4\right) \\
\underline{I}_4 &= \underline{y}_{34}\left(\underline{V}_4 - \underline{V}_3\right) + \underline{y}_{44}\underline{V}_4
\end{aligned}
$$

- Note that obviously $\underline{y}_{ij} = \underline{y}_{ji}$, thus $\underline{\mathbf{Y}}$ is symmetric.

The basics

Admittance matrix – an example

- Now start from the KCLs

$$
\begin{aligned}
\underline{I}_1 &= \underline{y}_{12}\left(\underline{V}_1-\underline{V}_2\right)+\underline{y}_{13}\left(\underline{V}_1-\underline{V}_3\right)\\
\underline{I}_2 &= \underline{y}_{12}\left(\underline{V}_2-\underline{V}_1\right)+\underline{y}_{23}\left(\underline{V}_2-\underline{V}_3\right)\\
\underline{I}_3 &= \underline{y}_{23}\left(\underline{V}_3-\underline{V}_2\right)+\underline{y}_{13}\left(\underline{V}_3-\underline{V}_1\right)+\underline{y}_{34}\left(\underline{V}_3-\underline{V}_4\right)\\
\underline{I}_4 &= \underline{y}_{34}\left(\underline{V}_4-\underline{V}_3\right)+\underline{y}_{44}\underline{V}_4
\end{aligned}
$$

and express $\underline{\mathbf{Y}}$:

$$
\begin{bmatrix}\underline{I}_1\\ \underline{I}_2\\ \underline{I}_3\\ \underline{I}_4\end{bmatrix}
=
\begin{bmatrix}
\underline{y}_{12}+\underline{y}_{13} & -\underline{y}_{12} & -\underline{y}_{13} & 0\\
-\underline{y}_{12} & \underline{y}_{12}+\underline{y}_{23} & -\underline{y}_{23} & 0\\
-\underline{y}_{13} & -\underline{y}_{23} & \underline{y}_{13}+\underline{y}_{23}+\underline{y}_{34} & -\underline{y}_{34}\\
0 & 0 & -\underline{y}_{34} & \underline{y}_{34}+\underline{y}_{44}
\end{bmatrix}
\begin{bmatrix}\underline{V}_1\\ \underline{V}_2\\ \underline{V}_3\\ \underline{V}_4\end{bmatrix}
$$

- Verify the rule and convince yourselves (30 seconds):

$$
\underline{\mathbf{Y}} = [\underline{Y}_{ij}] = \begin{cases}
\underline{y}_{ii} + \sum_{j=1, j\neq i}^{n_B} \underline{y}_{ij} & \text{for the diagonal elements,}\\
-\underline{y}_{ij} & \text{for the other elements,}
\end{cases}
$$

- The admittance matrix can also be used to compute the power injected in all busses if voltages are known, as apparently

$$\underline{V} = \underline{\mathbf{Y}}^{-1}\underline{I}$$

where $\underline{V}$ and $\underline{I}$ are respectively the vectors of bus voltages and *injected* currents; matrix $\underline{\mathbf{Y}}^{-1}$ is also called the network (or nodal, or bus) *impedance* matrix, and denoted by $\underline{\mathbf{Z}}$.

- Therefore, knowing the bus voltage phasors, one can obtain the complex power injected at each bus as

$$\underline{S} = \underline{V} \circ \underline{I}^* = \underline{V} \circ (\underline{\mathbf{Y}}\,\underline{V})^*$$

where, remember, $\circ$ denotes the Schur product, and * the complex conjugate.

- Let us see here too an example, that will lead us to formulate the LF problem.

- Consider a network with two busses, and take $\underline{V}_1$ as phase reference (i.e., assume its phase is zero). Denote by δ_{21} the difference between the phases of $\underline{V}_2$ and $\underline{V}_1$ – i.e., the phase of $\underline{V}_2$ is δ_{21} – and go for Maxima:

```
Y : matrix([G11-%i*B11,G12-%i*B12],[G12-%i*B12,G22-%i*B22]);
V : matrix([V1],[V2*(cos(d21)+%i*sin(d21))]);
I : Y.V;
S : V*conjugate(I);
P : ratsimp(realpart(S));
Q : ratsimp(imagpart(S));
```

- We obtain for the injected (active and reactive) powers

$$\begin{aligned}
P_1 &= G_{11} V_1^2 + V_1 V_2 \left(G_{12} \cos\delta_{21} + B_{12} \sin\delta_{21} \right) \\
Q_1 &= B_{11} V_1^2 + V_1 V_2 \left(-G_{12} \sin\delta_{21} + B_{12} \cos\delta_{21} \right) \\
P_2 &= G_{22} V_2^2 + V_1 V_2 \left(-G_{12} \cos\delta_{21} - B_{12} \sin\delta_{21} \right) \\
Q_2 &= B_{22} V_2^2 + V_1 V_2 \left(G_{12} \sin\delta_{21} - B_{12} \cos\delta_{21} \right)
\end{aligned}$$

- In the example just treated, we ended up with four equations providing $P_{1,2}$ and $Q_{1,2}$, i.e., the (signed) active and reactive power injected at each of the two busses, based on knowledge of their voltage phasors (and of course of the network parameters).
- However, we could also view the problem in another way.
- Suppose for example that bus 1 is a Load (PQ) bus, and bus 2 a Generator (G) one. Suppose to know the active power demand from the load (remember the forecasts for tertiary control?) and that loads are managed so that the reactive demand is maintained within a prescribed power factor. Suppose, in one word, to know P_1 and Q_1.
- Suppose then that the *active* power generation at bus 2 is controlled (remember how P_m was managed to match P_e via power/frequency control?) and the same is true for the voltage *magnitude* V_2—not for the phase, as this is the means to release power. Actually we do not deal here with voltage control, leaving the matter to detailed courses, so just assume the job done.
- Again, take the phase of $\underline{V}_1$ ("the network" as opposed to the generator) as reference, and – remember – assume ideal (or almost ideal) frequency control.

- Given all the above, we are dealing again with the equations

$$\begin{aligned} P_1 &= G_{11} V_1^2 + V_1 V_2 \left(G_{12}\cos\delta_{21} + B_{12}\sin\delta_{21}\right) \\ Q_1 &= B_{11} V_1^2 + V_1 V_2 \left(-G_{12}\sin\delta_{21} + B_{12}\cos\delta_{21}\right) \\ P_2 &= G_{22} V_2^2 + V_1 V_2 \left(-G_{12}\cos\delta_{21} - B_{12}\sin\delta_{21}\right) \\ Q_2 &= B_{22} V_2^2 + V_1 V_2 \left(G_{12}\sin\delta_{21} - B_{12}\cos\delta_{21}\right) \end{aligned}$$

  but in the three unknowns Q_2, δ_{21}, V_2. Note that also in the example the free quantities were in fact three, as V_1, V_2 and δ_{21} decided all of the rest.
- To determine all the nodal voltage phasors, thus, we could write
  - the balance equation for active and reactive power at bus 1 (PQ),
  - and the balance equation for active power at bus 2 (G).
- This is a nutshell-size example of LF problem. Let us now abstract and generalise.

- Given a network composed of
  - n_G generator (G) busses,
    where the (injected) active power and the voltage amplitude are known,
  - plus n_{PQ} load (PQ) busses,
    where the (drawn) active and reactive powers are known,
  - plus one bus, called the *slack* (S) bus,
    where the voltage amplitude and phase are known,

  determine all the voltage phasors (magnitudes and phases).

- Note: of course $n_G + n_{PQ} + 1$ equals the total number n_B of busses, but in the following we shall preferably count busses by type.

- Unknowns:
  - $2(n_G + n_{PQ} + 1)$, i.e., all voltage phasors' magnitudes and phases
  - minus n_G because at G busses the voltage amplitude is known
  - minus 2 because at the S bus both voltage amplitude and phase are known,

  $\Rightarrow$ for a total of $n_G + 2n_{PQ}$.

- Equations:
  - n_G balances of active power at G busses,
  - plus n_{PQ}, balances of active power at PQ busses
  - plus n_{PQ}, balances of reactive power at PQ busses,

  $\Rightarrow$ for a total of $n_G + 2n_{PQ}$.

- Solution:
  - the problem is not dynamic, apparently,
  - but at the same time highly nonlinear;
  - many numerical methods were proposed and are continuously studied,
  - ranging from standard ones (e.g., Newton-Raphson) through modifications of them to completely *ad hoc* ones (the matter is not addressed in this course).

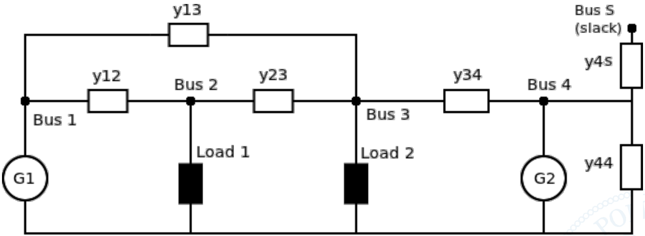
- All equations are active or reactive power balances, and take the form

$$
\begin{aligned}
P_i = & \sum_{j=1}^{n_B} V_i\, V_j\, \left(G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}\right), \\
Q_i = & \sum_{j=1}^{n_B} V_i\, V_j\, \left(G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij}\right),
\end{aligned}
$$

  where P_i and Q_i are respectively the active and reactive power injected or drawn at bus i, $G_{ij} - jB_{ij}$ is element (i, j) of the bus admittance matrix $\underline{\mathbf{Y}}$, and δ_{ij} the phase differences between the voltages at bus i and bus j, their amplitudes being V_i and V_j.
- As for the slack bus,
  - it can be viewed as a reference, *mutatis mutandis* pretty much like the ground when solving a circuit,
  - and additionally represent the connection to a larger – e.g., cross-national – network, viewed as a fixed phasor since that network can be assumed to have prevailing power w.r.t. the considered – e.g., national – one , like the considered one has w.r.t. any individual generator.

- Once primary/secondary control is in place and tertiary optimisation is done, use LF to check that no line is overloaded by also computing currents. If said check fails, modify the optimised solution to a suboptimal one "near" to the optimal but fulfilling the overload avoidance constraints.
- Express the mentioned overload avoidance conditions and plug them into tertiary optimisation as additional constraints. Note that this *significantly* complicates the optimisation problem as for constraint..
- On the same front, use LF to compute a transmission cost in terms of power lost over the lines, and plug this into tertiary optimisation. Also doing so complicates the optimisation, this time however by modifying the cost function.
- Let us proceed with an example.

- Write the LF equations for the network



- You have five minutes, then we do it together.

Load Flow

Example – solution

- First (although not strictly necessary, one could directly reason with its elements) write the bus admittance matrix:

$$
\underline{\mathbf{Y}} = \begin{bmatrix}
\underline{Y}_{11} & \underline{Y}_{12} & \underline{Y}_{13} & \underline{Y}_{14} & \underline{Y}_{1s} \\
\underline{Y}_{12} & \underline{Y}_{22} & \underline{Y}_{23} & \underline{Y}_{24} & \underline{Y}_{2s} \\
\underline{Y}_{13} & \underline{Y}_{23} & \underline{Y}_{33} & \underline{Y}_{34} & \underline{Y}_{3s} \\
\underline{Y}_{14} & \underline{Y}_{24} & \underline{Y}_{34} & \underline{Y}_{44} & \underline{Y}_{4s} \\
\underline{Y}_{1s} & \underline{Y}_{2s} & \underline{Y}_{3s} & \underline{Y}_{4s} & \underline{Y}_{ss}
\end{bmatrix}
$$

$$
= \begin{bmatrix}
\underline{y}_{12} + \underline{y}_{13} & -\underline{y}_{12} & -\underline{y}_{13} & 0 & 0 \\
-\underline{y}_{12} & \underline{y}_{12} + \underline{y}_{23} & -\underline{y}_{23} & 0 & 0 \\
-\underline{y}_{13} & -\underline{y}_{23} & \underline{y}_{13} + \underline{y}_{23} + \underline{y}_{34} & -\underline{y}_{34} & 0 \\
0 & 0 & -\underline{y}_{34} & \underline{y}_{34} + \underline{y}_{44} & -\underline{y}_{4s} \\
0 & 0 & 0 & -\underline{y}_{4s} & \underline{y}_{4s}
\end{bmatrix}
$$

Load Flow

Example – solution

- We have $n_G = 2$ and $n_{PQ} = 2$; $n_B = n_G + n_{PQ} + 1 = 2 + 2 + 1 = 5$.
- There are $n_G + 2n_{PQ} = 6$ unknowns: V_2, δ_{2s}, V_3, δ_{3s}, δ_{1s}, and δ_{4s}.
- The six equations are the two P balances at the G busses 1 and 4, the two P balances at the PQ busses 2 and 3, and the two Q balances at the same PQ busses. This yields

$$
\begin{aligned}
P_1 &= V_1^2\,(G_{12}+G_{13}) - V_1\,V_2\,(G_{12}\cos\delta_{12} + B_{12}\sin\delta_{12}) - V_i\,V_3\,(G_{13}\cos\delta_{13} + B_{13}\sin\delta_{13}) \\
P_4 &= V_4^2\,(G_{34}+G_{44}) - V_3\,V_4\,(G_{34}\cos\delta_{34} + B_{34}\sin\delta_{34}) - V_4\,V_s\,(G_{4s}\cos\delta_{4s} + B_{4s}\sin\delta_{4s}) \\
P_2 &= V_2^2\,(G_{12}+G_{23}) - V_1\,V_2\,(G_{12}\cos\delta_{12} + B_{12}\sin\delta_{12}) - V_2\,V_3\,(G_{23}\cos\delta_{23} + B_{23}\sin\delta_{23}) \\
Q_2 &= -V_2^2\,(B_{12}+B_{23}) - V_1\,V_2\,(G_{12}\sin\delta_{12} - B_{12}\cos\delta_{12}) - V_2\,V_3\,(G_{23}\sin\delta_{23} - B_{23}\cos\delta_{23}) \\
P_3 &= V_3^2\,(G_{13}+G_{23}+G_{34}) + V_1\,V_3\,(G_{13}\cos\delta_{13} + B_{13}\sin\delta_{13}) \\
&\quad + V_2\,V_3\,(G_{23}\cos\delta_{23} + B_{23}\sin\delta_{23}) + V_3\,V_4\,(G_{34}\cos\delta_{34} + B_{34}\sin\delta_{34}) \\
Q_3 &= -V_3^2\,(B_{13}+B_{23}+B_{34}) + V_1\,V_3\,(G_{13}\sin\delta_{13} - B_{13}\cos\delta_{13}) \\
&\quad + V_2\,V_3\,(G_{23}\sin\delta_{23} - B_{23}\cos\delta_{23}) + V_3\,V_4\,(G_{34}\sin\delta_{34} - B_{34}\cos\delta_{34})
\end{aligned}
$$

where diagonal-originated terms were put at the beginning of summations for readability (clearly $\sin\delta_{ii} = 0$ and $\cos\delta_{ii} = 1\ \forall\, i$).

Conclusions

- We now have a reasonably complete *panorama* of network control and optimisation, excluding just reactive power control, collapse avoidance and contingency management at large. This is enough for our purposes, since we have caught the concepts and so as to see any detail – taught in specialised courses – within a unitary framework.
- We shall thus proceed with a practice sessions, thereby concluding the "electric" part of the course.
- No homework, but remember the benefits of reviewing your notes.

Classroom practice

Load flow and its relationships with network control

Lessons to learn

i.e., just one slide for this lecture's objectives

- Become familiar with LF.
- Avoid the most common mistakes, or – better – ensure that the meaning of all elements is well understood.
- Envisage the relationships between LF and primary/secondary/tertiary network control.

Lecture 14 (2L)

Introduction to thermal systems

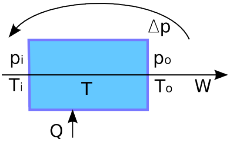
- Foreword and hypotheses
- Main components of heat networks and HVAC systems

Foreword and hypotheses

- We shall treat thermo-vector fluids (e.g., water flowing in heating elements) as incompressible, and with constant specific heat,
- and the same will be done for the air contained in buildings (pressure is practically the atmospheric one):
- thus, we shall not treat centralised air treatment systems like AHUs (Air Handling Units) and air-based heat distribution.
- Solid materials will come into play for containments, and here too simple descriptions (constant and uniform properties) will be used:
- always recall our system-level attitude.

- Fluid transport, motion and flow control (pipes, pumps, valves);
- heat exchangers (fluid/fluid and fluid/air);
- thermal machines (boilers, chillers, heat pumps);
- containment elements and associated thermal exchanges (walls, openings, conductive/convective/radiative exchanges);
- and – not in this lecture – control elements (sensors, actuators, controllers).
- Also, we shall take an *energy-based* approach, using electric equivalents when possible and refraining from the description of hydraulics, mechanics and so forth.
- Note: for simplicity, unidirectional fluid flow (quite realistic, anyway) will be assumed.

Main components of heat networks and HVAC systems



- Hydraulic equation (review, Δz in-out height difference, ρ fluid density):

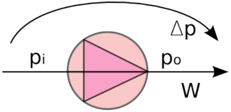
$$\Delta p = \frac{K_T}{\rho} w^2 - \rho g \Delta z$$

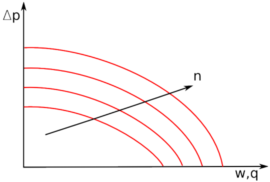
- Energy balance (T assumed uniform and equal to T_o, V pipe volume, c fluid specific heat):

$$\rho V c \dot{T}_o = c w (T_i - T_o) + Q$$

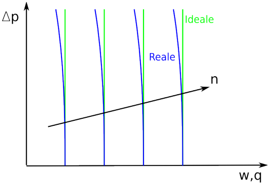
Fluid transport components

Pumps



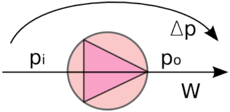


Centrifugal



Volumetric

- The command n prescribes the pump speed, typically expressed in rpm.
- Mass and volume flowrate (here proportional by ρ) are denoted by w and q, respectively.



- Centrifugal pump hydraulics (given the fluid):

$$\Delta p = H0(n) - H1(n)w^2$$

where, given a nominal rpm n_0 and correspondingly $H_0 = \overline{H}_0$ e $H_1 = \overline{H}_1$,

$$H0(n) = \overline{H}_0 \frac{n}{n_0}, \quad H1(n) = \overline{H}_1 \frac{n}{n_0}.$$

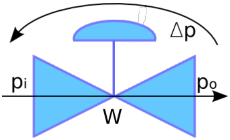
- Volumetric pump hydraulics (given the fluid):

$$w = Kn$$

where K is a characteristic parameter.

- As for thermal aspects, in both cases (neglecting mechanical heating)

$$T_o = T_i$$



- Hydraulic equation (review):

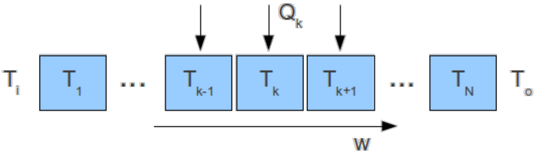
$$w = Cv_{max}\Phi(x)\sqrt{\Delta p}$$

- Energy balance (both storage and exchanges neglected):

$$T_o = T_i$$

Heat exchangers

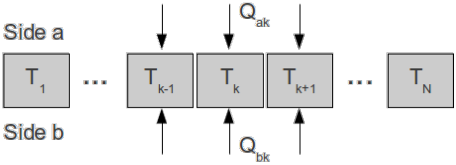
Fluid stream – lumped (FiniteVolumes) model



- For incompressible fluids, thermal equations are decoupled from hydraulics (constant density).
- Having decided the flow direction, the equation for each element of the FV description is thus

$$\rho c V_k \dot{T}_k = wcT_{k.1} - wcT_k + Q_k, \quad T_{-1} = T_i, \; T_o = T_N,$$

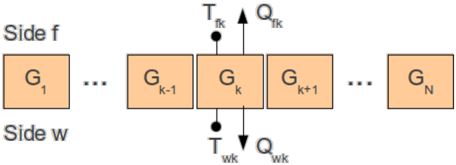
where the V_k are the lumps' volumes, often (for us, always) equal.



- Dividing the metal wall in the same way as the stream(s),

$$\rho_m c_m V_{mk} \dot{T}_k = Q_{ak} + Q_{bk}$$

  where the "$_m$" subscript stands for "metal".
- Axial conduction in the metal is neglected, which is realistic enough for our purposes.



- Finally comes the (convective) exchange element, that adopting again the same discretisation yields

$$Q_{fk} = -Q_{mk} = \gamma S_k (T_{fk} - T_{mk})$$

where the "$_f$" and "$_m$" subscript stands for "fluid" and "metal", respectively. The S_k are the lumps' surfaces and γ is the thermal exchange coefficient, either constant (as we shall assume) or dependent on the flow conditions.

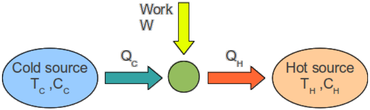
- The connections realise the heat exchanger configuration (e.g., co- or counter-current).

- For our purposes, we distinguish two types of machines:
  - those injecting heat into a fluid by combustion or analogous sources (e.g., solar radiation) that can possibly modulated (e.g., by operating a fuel valve of focusing/defocusing a mirror); these include boilers, thermal solar panels, and similar objects;
  - those employing work to transfer heat from a cold to a hot source; these include all types of heat pumps.
- At a system level, thermal machines can be represented as static relationships coupled to a simple (for us, first-order) dynamics to represent the internal storage, or in other words connected to the fluid "residence time".
- In such descriptions the fluid(s) see the machine operation as impressed heat rates: possible relationships of that rate with temperatures are represented as static characteristics in the machine model.

- Denoting by w the fluid flowrate, by w_f the fuel one, by HH its calorific power, by V the volume of contained fluid and by η_c a "combustion" efficiency, one can simply write with self-explanatory notation

$$\rho c V \dot{T}_o = wc(T_i - T_o) + w_f HH \eta_c(w, T_o)$$

  where $\eta_c(w, T_o)$, or just $\eta_c(w)$ depending on the detail level, provides the machine's efficiency curve.
- Suggestion: try to reformulate for a thermal solar captor where the captured radiative flux can be partialised.



- Statically, the heat rate balance is apparently $Q_H = Q_C + W$, where W can come e.g. from a compressor (for brevity we do not treat more articulated cases such as absorption cycles).
- Machines like that here schematised are statically described by the so called *Coefficient Of Performance* (COP), defined as "useful effect over needed work". Thus, since one may be interested in using the machine for heating or for cooling, we have

$$COP_{heat} = \frac{Q_H}{W}, \quad COP_{cool} = \frac{Q_C}{W}$$

- Observe that

$$\frac{Q_H}{W} = \frac{Q_C + W}{W} \Rightarrow COP_{heat} = COP_{cool} + 1.$$

- If the machine operates at the maximum theoretical (Carnot) efficiency,

$$\frac{Q_H}{T_H} = \frac{Q_C}{T_C}$$

- Thus, theoretical (Carnot) values for COP_{heat} and COP_{cool} can be defined as

$$COP^C_{heat} = \frac{T_H}{T_H - T_C}, \quad COP^C_{cool} = \frac{T_C}{T_H - T_C},$$

  where, remember, *absolute* (Kelvin) temperatures are to be used.

- To represent real machines, efficiencies are introduced, hence

$$COP_{heat} = \eta_h \frac{T_H}{T_H - T_C}, \quad COP_{cool} = \eta_c \frac{T_C}{T_H - T_C}, \qquad 0 < \eta_{h,c} < 1,$$

  where η_h and η_c can be considered constant (as we shall do) or be made dependent on temperatures.

- To obtain a simple (system-level) dynamic model, associate two thermal capacities $C_{H,C}$ to the hot and cold sources (i.e., considering the classical refrigerator cycle as a representative example, to the condensed and the evaporator, respectively). This, together with the previous COP considerations, yields for the heating case

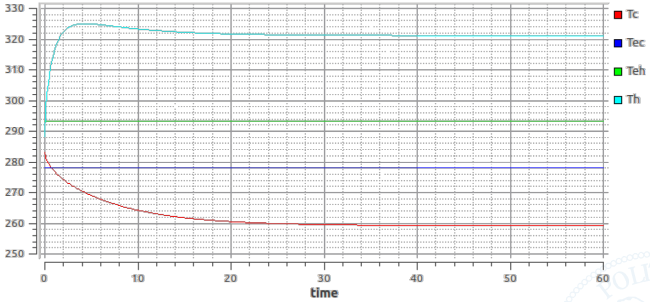
$$
\begin{aligned}
C_H \dot{T}_H &= Q_H + Q_{Heh} \\
C_C \dot{T}_C &= Q_C + Q_{Cec} \\
COP_{heat} &= \eta_h \frac{T_H}{T_H - T_C} \\
Q_H &= COP_{heat} W \\
Q_H &= Q_C + W
\end{aligned}
$$

where Q_{Heh} and Q_{Cec} are the heat rates entering the H and C sources from the environments to which either of the two is exposed (most frequently, by convection); W is here an input.

- Modelica example:

```
model coph
  parameter Real Wmax=1000; parameter Real Cc=1000; parameter Real Ch=200;
  parameter Real Gec=100;   parameter Real Geh=100;
  Real Tc(start=273.15+10); // MEMENTO: Kelvin temperatures trhoughout!
  Real Th(start=273.15+15);
  Real Tec,Teh,Qc,Qh,Qcec,Qheh,W,COPh,eta,cmd;
equation
  Ch*der(Th) =  Qh - Qheh;
  Qheh       =  Geh*(Th-Teh);
  Cc*der(Tc) = -Qc + Qcec;
  Qcec       =  Gec*(Tec-Tc);
  COPh       =  Th/(Th-Tc)*eta;
  Qh         =  COPh*W;
  Qh         =  Qc+W;
  W          =  Wmax*cmd; // W is an input, cmd its command
  Tec        =  273.15+5;  // Boundary conditions
  Teh        =  273.15+20;
  eta        =  0.6;
  cmd        =  0.9;
end coph;
```

- Simulation results example:



- Homework 1: change parameters (thermal capacities, max power, coefficients for convective exchanges, efficiency); observe how the results are affected, and interpret.
- Homework 2: reformulate for a cooling case.

Containment elements and associated exchanges

- Containment elements are simple conduction ones.
- Such elements are possibly (and in fact most frequently) composed of a series of layers of different materials (i.e., with different specific heats and conductivities)...
- ...so that, anticipating some concepts analysed later on, their electric equivalent is a series of RC cells.
- Glazing may require specific modelling (for example, windows often need to account for the thermal power transmitted by radiation), but we do not deal with such details in this course.

Containment elements and associated exchanges

- A specific case is the effect of air renovation, on which we conversely spend some words.
- Denoting by w_r the renovation air flow rate, and with T_i and T_e the internal and external temperatures, respectively, we have

$$\begin{aligned} Q_{e\rightarrow i} &= w_r c_a T_e \\ Q_{i\rightarrow e} &= w_r c_a T_i \end{aligned}$$

  where c_a is the air specific heat.
- Hence, summing the above with the correct signs, the net external-to-internal heat rate is

$$Q_{ei} = G_{ei}(T_e - T_i), \quad G_{ei} = w_r c_a$$

  where G_{ei} plays the role of an equivalent thermal conductance, governed by the renovation flowrate.

Lecture 15 (2L)

Heat networks and HVAC systems

- Electric equivalents
- System-level modelling for control

Electric equivalents

- Energy storage $\leftrightarrow$ capacitor, and *under our hypotheses* .
  - voltage $[V] \leftrightarrow$ temperature $[^\circ K]$,
  - current $[A] = [C/s] \leftrightarrow$ thermal power or "heat rate" $[W] = [J/s]$
  - electric capacity $[F] = [C/V] \leftrightarrow$ thermal capacity $[J/^\circ K]$;
- dimensional consistency check:

$$
\begin{array}{cccc|cccc}
I & = & C_E & \dfrac{dV}{dt} & Q & = & C_T & \dfrac{dT}{dt} \\
[C/s] & = & [C/V] & [V/s] & [J/s] & = & [J/^\circ K] & [^\circ K/s]
\end{array}
$$

- For solids or incompressible fluids, both with constant specific heat c $[J/kg^\circ K]$ – our hypotheses – the thermal capacity C_T (we shall hereinafter drop the subscript wherever possible) is expressed as $\rho c V$, where ρ is the solid or fluid density, and V the considered control volume.

- Energy transfer without mass transfer, proportional to a temperature *difference* $\leftrightarrow$ resistor and
  - electric conductance $[S]=[A/V] \leftrightarrow$ thermal conductance $[W/^\circ K]$;
- dimensional consistency check:

$$\begin{array}{cccc|cccc} I & = & G_E & \Delta V & Q & = & G_T & \Delta T \\ [A] & = & [A/V] & [V] & [W] & = & [W/^\circ K] & [^\circ K] \end{array}$$

- This covers (simple) conduction and convection; radiation involves the fourth power of temperatures, but most often for our cases can be approximated with a prescribed heat rate—i.e., a current generator.
- The thermal conductance G (again, subscript dropped from now on) is typically expressed as γS, where S is the boundary surface of the considered control volume and γ $[W/m^{2\circ}K]$ is called "thermal exchange coefficient".

- Energy transfer associated to (fluid) mass transfer $\leftrightarrow$ current generator commanded by a (source) temperature, with a proportional law where the proportionality coefficient is the involved mass flowrate times the fluid specific heat;
- dimensional consistency check:

$$
\begin{array}{cccc|cccc}
I & = & k & V_{src} & Q & = & wc & T_{src} \\
[C/s] & = & [C/Vs] & [V] & [J/s] & = & [kg/s][J/kg^\circ K] & [^\circ K]
\end{array}
$$

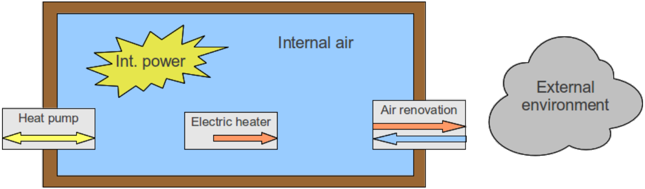
- Observe that this is different from energy transfer without mass transfer, as only one temperature – not a difference – is involved; also, here changing the sign of w means "the source is on the other side" (but for our problems we do not need to account for flow reversal).
- Recall that we assume hydraulics to be decoupled from thermal phenomena, hence when dealing with the latter we can consider flowrates to be known.

System-level modelling for control

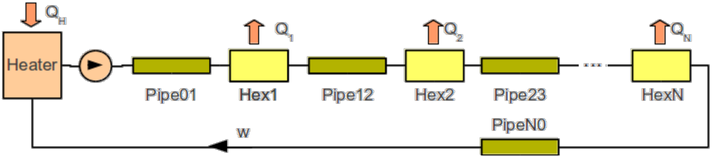
- Purpose:
  - obtain simple models catching the system behaviour without delving into individual component details (as usual).
- General ideas:
  - limit the scope to energy generation/transfer/storage;
  - adopt the electric equivalent approach just sketched,
  - ether explicitly or by just using their equations' structuring.
- The easiest way to understand the matter is to work out a couple of simple examples, which by the way involves some Modelica practice.

System-level modelling for control

Example 1



- Single thermal zone with
  - exchanging walls,
  - air renovation by direct connection to the external environment,
  - a reversible heat pump for heating or cooling,
  - an additional electric heater,
  - and internally generated power (people, computers, and so on).



- Simple heat network with
  - one centralised heater,
  - multiple heat exchangers modelled as prescribed thermal loads (in turn serving some building temperature control),
  - and an idealised primary loop flowrate control.

- Example 1 (thermal zone):
  - one temperature controller with signed power output (+ heat, - cool) and possibly a dead zone;
  - split-range output using the heat pump when cooling and first the pump, then the electric device (less efficient) when heating;
  - if possible, feedforward compensation for the external temperature, the renovation flowrate, and the internally generated power.
- Example 2 (heat network):
  - flowrate control;
  - heater outlet temperature control;
  - computation of the two set points so as to satisfy all the thermal loads while minimising piping losses.

- We apparently need *control structures*, which we shall review and apply in the next lecture.
- No homework, but please review your notes. Next time, practice session

Lecture 16 (2P)

Classroom practice

Modelling and basic control of heat networks and HVAC systems

Lessons to learn

i.e., just one slide for this lecture's objectives

- Work out some more examples than those shown last time.
- Apply standard control techniques, basically of the single-loop type.
- Envisage the necessary extensions as for the control scheme structuring.

Lecture 17 (2L)

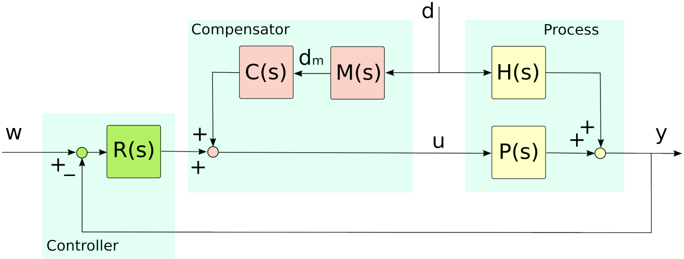
Main control structures for energy systems

- **Control structures *stricto sensu***
- **Actuation schemes**

- We shall now review the major structures of interest, i.e.,
  - feedforward compensation,
  - cascade control,
  - multivariable control with decoupling (in the 2×2 case, generalisation is straightforward),
  - Smith predictor,
  - Internal Model Control (IMC).
- We shall also review some relevant *actuation* schemes, i.e.,
  - Time Division Output (TDO),
  - split range,
  - daisy-chaining.

Control structures *stricto sensu*

Feedforward compensation



- Purpose: reduce the influence on the controlled varuable y of a *measurable* disturbance $d(t)$ acting on the forward path of a loop.
- How: by computing $C(s)$ so that the transfer function d to y be zero, i.e.,

$$\frac{H(s)+M(s)C(s)P(s)}{1+R(s)P(s)}=0, \quad \Rightarrow C(s)=-\frac{H(s)}{M(s)P(s)}$$

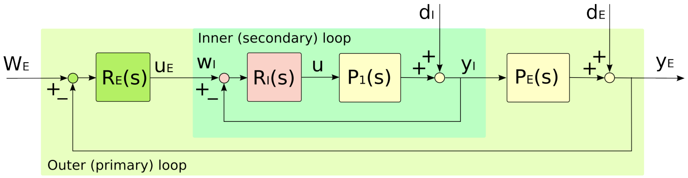
- The so found compensator is "ideal", and will be termed $C_{ID}(s)$, as

$$C_{ID}(s) = -\frac{H(s)}{M(s)P(s)}$$

  - may have more zeroes than poles (not realisable),
  - and/or have RHP poles (i.e., produce critical cancellations).

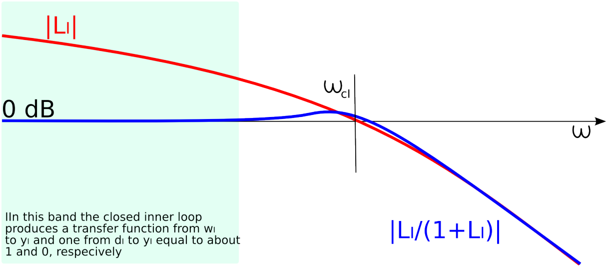
- In such cases one has to obtain from $C_{ID}(s)$ a *real* compensator $C_R(s)$
  - omitting zeroes and/or adding poles,
  - and in any case not introducing RHP poles.
- This will yield a compensation "valid up to a certain frequency", namely that for which $C_R(j\omega)$ starts to differ "significantly" from $C_{ID}(j\omega)$, accounting for *both* magnitude and phase.

Cascade control



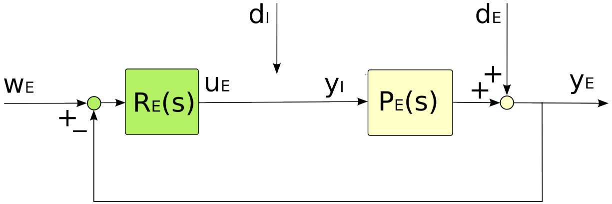
- Purpose: mitigate the effects of a disturbance (d_I) the effects of which appear on some *measurable* process variable (y_I) responding to that disturbance "before" – in a dynamic sense – the primary controlled variable (y_E).
- How: by closing a "fast" inner (internal, secondary) loop so as to hide both the dynamics of P_I and the effects of d_I to the outer (external, primary) one.
- Two remarks:
  - the scheme has the inherent cost of measuring y_I,
  - and is hardly of any use (i.e., a single-loop one could do the job as well) in the absence of a significant d_I.

- Explanation of the scheme operation in the frequancy domain:



where $L_I = R_I P_I$.

- In practice, indicating by ω_{cI} and ω_{cE} the critical frequencies of the inner and the outer loop, respectively, a minimum bandwidth separation of 0.5–1 decade is advised.
- If this is accomplished, it is possible to (approximately but reliably) compute ω_{cE} as if the external (open) loop transfer function were just $R_E P_E$.



- For the synthesis, thus, one can refer for the external loop to the scheme above.
- Overall, this leads to determining
  - R_I based on P_I only
  - and R_E based on P_E only (preserving of course the required band separation).

Multivarlable control with decoupling

- Purpose: address *square* MIMO *interacting* processes (a control input does not influence only one controlled output).
- How: by taking a two-step approach, namely
  - forst prepending to the process a *decoupler* so that the cascade of the teo be diagonal,
  - and then closing one SISO loop per variable, synthesised with the known techniques.

- Note: as anticipated we treat the 2×2 scheme, i.e., two inputs u_1, u_2 and two outputs y_1, y_2; generalising to the generic $n \times n$ case is straightforward.

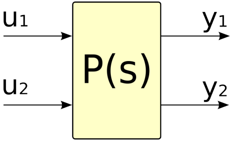
- A 2×2 MIMO process is described in the LTI framework as

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}$$

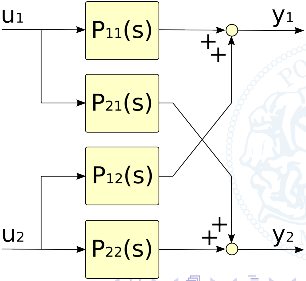
i.e., by the transfer matrix

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix}$$

- In terms of block diagrams this means

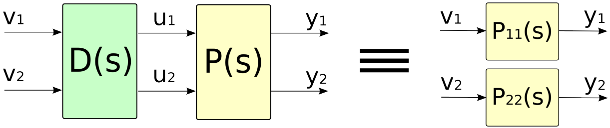


$$\begin{cases} Y_1 = P_{11}U_1 + P_{12}U_2 \\ Y_2 = P_{21}U_1 + P_{22}U_2 \end{cases} \Rightarrow$$



Multivarlable control with decoupling
The decoupling block

- The purpose of $D(s)$ is to realise the equivalence indicated below:



i.e., setting $Y' = [Y_1\ Y_2]$, $U' = [U_1\ U_2]$ e $V' = [V_1\ V_2]$,

$$Y = PU = PDV = \begin{bmatrix} P_{11}(s) & 0 \\ 0 & P_{22}(s) \end{bmatrix} V$$

- Therefore, $D(s)$ is determined as

$$D(s) = P^{-1}(s) \begin{bmatrix} P_{11}(s) & 0 \\ 0 & P_{22}(s) \end{bmatrix}$$

- Interpretation:

$$D = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}^{-1} \begin{bmatrix} P_{11} & 0 \\ 0 & P_{22} \end{bmatrix}$$

hence

$$\begin{aligned} D^{-1} &= \left(\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}^{-1} \begin{bmatrix} P_{11} & 0 \\ 0 & P_{22} \end{bmatrix} \right)^{-1} \\ &= \begin{bmatrix} P_{11} & 0 \\ 0 & P_{22} \end{bmatrix}^{-1} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \\ &= \frac{1}{P_{11}P_{22}} \begin{bmatrix} P_{22} & 0 \\ 0 & P_{11} \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \\ &= \frac{1}{P_{11}P_{22}} \begin{bmatrix} P_{11}P_{22} & P_{12}P_{22} \\ P_{11}P_{21} & P_{11}P_{22} \end{bmatrix} = \begin{bmatrix} 1 & P_{12}/P_{11} \\ P_{21}/P_{22} & 1 \end{bmatrix} \end{aligned}$$

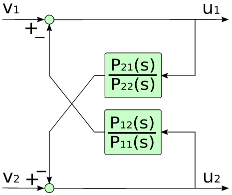
- Carrying on, $U = DV \Rightarrow V = D^{-1}U$, thus

$$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} 1 & \frac{P_{12}}{P_{11}} \\ \frac{P_{21}}{P_{22}} & 1 \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix} \Rightarrow \begin{cases} V_1 = U_1 + \frac{P_{12}}{P_{11}} U_2 \\ V_2 = \frac{P_{21}}{P_{22}} U_1 + U_2 \end{cases}$$

- In synthesis, then

$$\begin{cases} U_1 = V_1 - \frac{P_{12}}{P_{11}} U_2 \\ U_2 = V_2 - \frac{P_{21}}{P_{22}} U_1 \end{cases}$$

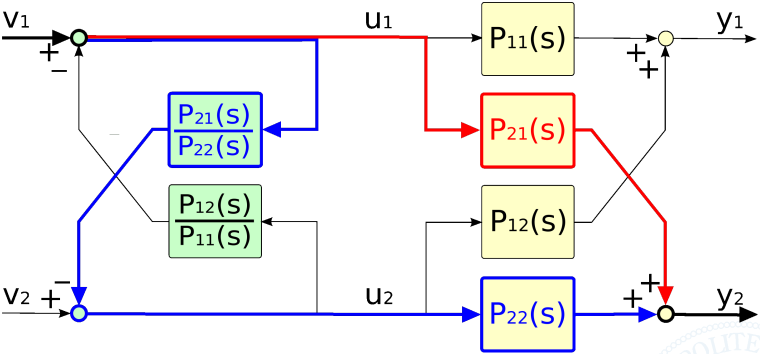
and the decoupler is described by the block diagram



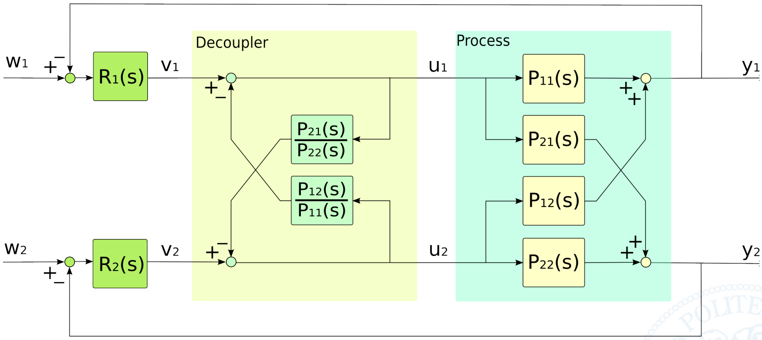
whence the frequently encountered name "backward decoupling".

Multivarlable control with decoupling

The decoupling block

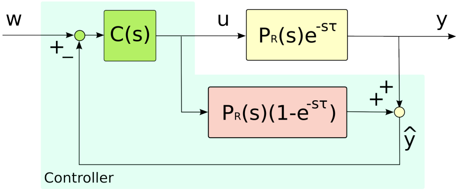


- The scheme above shows the backward decoupler operation by evidencing how it nullifies the net signal path from v_1 to y_2 (the sum of the read and tha blue one); of course the same is true for the symmetric path.
- Note; the same feasibility/stability ossues shown for feedfirward compensation may arise, requiring to use approximated decoupling blocks and thus limiting the band where decoupling is effective.



- The two controllers $R_1(s)$ and $R_2(s)$ are designed with known SISO techniques, as if one were dealing with two independent processes having transfer function $P_{11}(s)$ and $P_{22}(s)$, respectively

Smith predictor



- Purpose: address cases where the process has so large a delay that obtaining a certain stability degree (e.g., a desired phase margin) requires to reduce performance (e.g., response speed) unacceptably.
- How: by observing that in the scheme above

$$\frac{\widehat{Y}(s)}{U(s)} = P_R(s)$$

where the transfer function $P_R(s)$ is assumed rational.
- Block $C(s)$ can thus be synthesised with known methods, accounting only for the "rational dynamics" of the process.

Smith predictor

- The scheme immediately yields

$$\frac{\widehat{Y}(s)}{W(s)} = \frac{C(s)P_R(s)}{1 + C(s)P_R(s)}$$

- Additionally, the "regulator" in the same scheme is equivalent to a feedback one with transfer function

$$R(s) = \frac{C(s)}{1 + C(s)P_R(s)(1 - e^{-s\tau})}$$

thus

$$\begin{aligned}
\frac{Y(s)}{W(s)} &= \frac{R(s)P_R(s)e^{-s\tau}}{1 + R(s)P_R(s)e^{-s\tau}} \\
&= \frac{\frac{C(s)}{1+C(s)P_R(s)(1-e^{-s\tau})} P_R(s)e^{-s\tau}}{1 + \frac{C(s)}{1+C(s)P_R(s)(1-e^{-s\tau})} P_R(s)e^{-s\tau}} = \ldots = \frac{C(s)P_R(s)e^{-s\tau}}{1 + C(s)P_R(s)}
\end{aligned}$$

Smith predictor

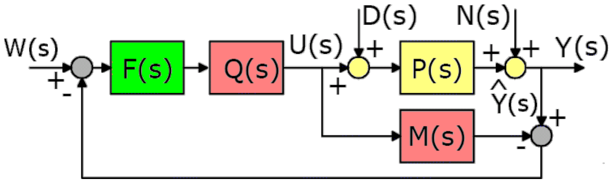
- In synthesis, then,

$$\frac{Y(s)}{W(s)} = \frac{\widehat{Y}(s)}{W(s)} e^{-s\tau}$$

which means that synthesising $C(s)$ based on $P_R(s)$ and using the Smith predictor scheme, the obtained behaviour of y is the same as that of $\widehat{y}$, just delayed by τ.
- Caveat:
  - the model has to be "more precise" than needed for mere feedback control
  - and disturbances need not to be too significant,

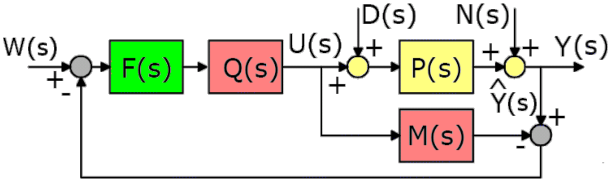
otherwise $\widehat{y}$ ceases to be a good prediction of y, to the detriment of the scheme operation.

Internal Model Control



- Purpose: obtain the regulator directly from a process model and one of the desired closed-loop dynamics.
- Most typical motivations:
  - standard synthesis method (e.g., used to derive PI/PID tuning rules);
  - easy adaptation of the controller to a (re-)estimated model.
- How: by adopting the scheme above, which we shall now analyse.

- Consider the block diagram



where

- $P(s)$ is the transfer function of the process under control, which we assume to be asymptotically stable (thus excluding integrating processes),
- $M(s)$ is the process model,
- $Q(s)$ and $F(s)$ are asymptotically stable transfer function, at this stage arbitrary,
- W, D and N are the set point, a load disturbance, and a measurement noise,
- Y and $\widehat{Y}$ are the true and nominal controlled variables, i.e., the outputs of the process and the model, respectively.

Internal Model Control

- Note (keep the previous slide at hand) that the feedback signal is $Y - \widehat{Y}$, which motivates the method's name in that the regulator contains a model of the process *explicitly*.
- Suppose that the model is perfect and there are neither disturbances nor noise. In that case the scheme is open-loop, and $T(s) := \frac{Y(s)}{Y^{\circ}(s)} = F(s)Q(s)M(s)$.
- Suppose now also that it is possible to choose $Q(s)$ as the exact inverse of $M(s)$, i.e. of $P(s)$. In that case $T(s) = F(s)M^{-1}(s)M(s) = F(s)$, thus the transfer function from set point to controlled variable can be chosen arbitrarily.
- Finally, suppose that $d \neq 0$ while still $M(s) = P(s)$, $n = 0$ and $Q(s) = M^{-1}(s)$.
  Being $Y(s)/D(s) = (1 - F(s)Q(s)M(s))/(1 + F(s)Q(s)(P(s) - M(s))$, if $F(s) = 1$ d is rejected completely; otherwise, it is rejected asymptotically provided that $F(0) = 1$.

- The IMC regulator is equivalent to the feedback one given by

$$R(s) = \frac{F(s)Q(s)}{1 - F(s)Q(s)M(s)}.$$

- The IMC synthesis method in the general case is a two-step procedure:
  - first $Q(s)$ is chosen as as an approximated inverse of $M(s)$, namely that of its minimum-phase part;
  - then, the low-pass filter $F(s)$, called “IMC filter”, is introduced; for simplicity, $F(s)$ is very often chosen of the first order and (of course) with unity gain, i.e. $F(s) = 1/(1 + s\lambda)$.
- In this case, λ can be interpreted as the closed-loop time constant of the control system if $P(s) = M(s)$, and $P(s)$ is minimum-phase. More in general, it can be thought of as the dominant closed-loop *desired* time constant.

Actuation schemes

Time Division Output

- Purpose: make an on/off actuator behave like a modulating one.
- Most typical motivations:
  - modulating high-power actuators may be impractical or even impossible;
  - even in the absence of the above problem, operating the actuator not at 100% may reduce its efficiency.
- How: by deciding an *actuation period* T_a, small w.r.t. the process dynamics' time scale, taking this as the sampling time for the (digital) controller, and having the control signal $u \in [0, 1]$ provide the actuator activation's duty cycle.

- Example: with $T_a = 10\,s$, $u = 0.6$ means that in the sampling period the actuator will be on for $6\,s$ and then off for $4\,s$.

Split range

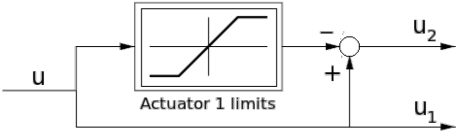
- Purpose: make two actuators behave like a single one by having each of them act in a different range of the control variable (whence the name).
- Most typical motivation:
  - thermal system where a single controller uses two actuators, one for heating and one for cooling.
- How: denoting by $u_1 \in [0, 1]$ and $u_2 \in [0, 1]$ the two actuators and supposing – without loss of generality – that the transition happens for $u = 0$, where $u \in [-1, 1]$ is the controller output, by simply setting

$$u_1 = \begin{cases} u & u \geq 0 \\ 0 & u < 0 \end{cases} \qquad u_2 = \begin{cases} 0 & 0 \geq 0 \\ -u & u < 0 \end{cases}$$

- Note: sometimes a *dead zone* is introduced around $u = 0$ to avoid switching in and out the two actuators too frequently.

Daisy chaining

- Purpose: have several actuators activated in sequence (the $i+1$-th starting to operate when the i-th has reached its maximum action).
- Most typical motivation:
  - start with the most economic actuatoe (e.g., a heat pump) and have a less economic one (e.g., an electric heater) intervene only if the first one is not sufficient.
- How: by using the diagram



  that easily generalises to an arbitrary number of actuators.

- Note:here too dead zones are sometimes introduced, for the same reason mentioned above.

Lecture 18 (2P)

Classroom practice

Control structures applied to thermal systems control

Lessons to learn

i.e., just one slide for this lecture's objectives

- Apply some of the analysed control structures to thermal systems.
- Abstract a more general *modus operandi* for control structuring, also in a view to addressing multi-domain problems.